ICAL ROOM
AL LIBRARY
OF MICH.
NOV 10 1924

Vol. 198 NOVEMBER, 1924 No. 5

1826

Journal of The Franklin Institute

1924

EDITOR, GEORGE A. HOADLEY, Sc.D.
ASSISTANT EDITOR, ALFRED RIGLING

Associate Editors

JOSEPH S. AMES, PH.D. — ARTHUR L. DAY, Sc.D. — R. B. MOORE, Sc.D.
WILDER D. BANCROFT, PH.D. — A. S. EVE, F.R.S. — MAJ. GEN. G. O. SQUIER, PH.D.
R. J. BERG, Sc.D. — PAUL D. FOOTE, PH.D. — W. F. G. SWANN, D.Sc.
BRIG. GEN. JOHN J. CARTY, LL.D. — W. J. HUMPHREYS, PH.D. — D. W. TAYLOR, LL.D.
E. G. COKER, F.R.S. — A. E. KENNELLY, Sc.D. — A. F. ZAHM, PH.D.
ALLERTON S. CUSHMAN, PH.D. — C. E. K. MEES, D.Sc. — JOHN ZELENY, PH.D.
RALPH MODJESKI, D.Eng.

Committee on Publications

GEORGE D. ROSENGARTEN, CHAIRMAN
GILBERT ALLEMAN — CHARLES E. BONINE — GEORGE A. HOADLEY — E. H. SANBORN

CONTENTS

Address by Elihu Thomson, Sc.D., on the Occasion of the Observance of the Centenary of the Founding of The Franklin Institute 581

The Fifth Estate 599
ARTHUR D. LITTLE

The Carbon Atom in Crystalline Structure 615
SIR WILLIAM HENRY BRAGG

Preliminary Measurement of the Velocity of Light 627
A. A. MICHELSON

Studies in Photographic Sensitivity 629
S. E. SHEPPARD, R. P. WIGHTMAN AND A. P. H. TRIVELLI

Heaviside's Operators in Engineering and Physics 647
ERNST JULIUS BERG

Notes from the U. S. Bureau of Standards 703

Notes from the U. S. Bureau of Chemistry 709

Notes from the U. S. Bureau of Mines 711

The Franklin Institute Notes 713

Book Reviews 718

Publications Received 722

Current Topics 625, 626, 646, 702, 708, 712, 723

Published Monthly by

THE FRANKLIN INSTITUTE OF THE STATE OF PENNSYLVANIA, PHILADELPHIA

SIX DOLLARS PER YEAR (Foreign Postage Additional) SINGLE NUMBERS, SIXTY CENTS
Entered at the Post Office at Philadelphia, Pa., as second-class matter. Copyright, 1924, by THE FRANKLIN INSTITUTE

Acceptance for mailing at special rate of postage provided for in section 1103, Act of October 3, 1917, authorized on July 3, 1918.

EMENTS.

CO.

ETS

Y.

O.

Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 198 NOVEMBER, 1924 No. 5

ADDRESS BY ELIHU THOMSON, Ph.D., Sc.D., ON THE OCCASION OF THE OBSERVANCE OF THE CENTENARY OF THE FOUNDING OF THE FRANKLIN INSTITUTE.*

His Honor the Mayor, Mr. President, Distinguished Guests, Ladies and Gentlemen:

As Honorary Chairman of the Centenary Celebration Committee of The Franklin Institute I have been appointed—with no subject assigned—to address you, a condition which permits me to deal generally with the past period of one hundred years of the life of the Institute. Before doing this I would like to refer briefly to the significance of such an occasion as this, bringing together so much of scientific talent and eminence in research, evidenced in the names of the speakers who are to address the sections according to the carefully prepared program. The Institute is indeed fortunate in having on the list not only many of the most distinguished in our own land, but also exceptional representation from over the seas, the whole forming a body of scientists and engineers seldom brought together on such an occasion. The impressive list of representatives of universities and colleges everywhere, the body of leaders in science who are listed as delegates from the learned and professional societies, and the numerous company of those from the great industrial organizations, together form a gathering of high standing in science, in

* Delivered in the Walnut Street Theatre, September 17, 1924.

(Note.—The Franklin Institute is not responsible for the statements and opinions advanced by contributors to the JOURNAL.)

COPYRIGHT, 1924, by THE FRANKLIN INSTITUTE.

research and in the technical work which characterizes the age in which we live.

One might wish to be able to add that the age is equally well characterized by the application, in all the activities of life of at least a semblance, of the scientific method of discerning truth from falsity. Alas, there is a strong survival of prejudice, of bigotry, of mediæval ignorance in relation to the simple facts of life and existence in this world of ours; a clouding of mind only to be removed by more universal teaching of scientific principles, not alone in physics and chemistry, but in such subjects as biology, and its kindred branches, in economics, even also in politics and government. The advanced medical investigator still has to deal with the unreasoning prejudice of the anti-vivisectionist, it may be—or vaccinationist—or other anti what not, and the usual quackery of the fanciful names for fanciful practice. The naturalist is even at this day threatened in his conclusions and teachings, by the anti-evolutionist and his kin.

Fortunately, the more fundamental studies of physics and chemistry and the mechanic arts are not so banned or obstructed. The beneficial results are more immediate or direct, and less likely to cast doubt on the truth of dogma of any kind. Yet we have among us many who have blamed scientific developments for much of the terrible war we have so recently passed through, forgetting that a simple, short bar of that most useful of all metals, steel, can, by the simple act of sharpening one end thereof, be converted into a very dangerous weapon; but without the hand and mental resolve to wield it—how harmless. We will eliminate war, that horrible spectre which menaces civilization, when we, through understanding, unite the souls of men in the conquest of nature by science, which holds out limitless possibilities in every direction and when we obliterate greed, dishonor, bitterness, even the bitterness of religious differences, from men's minds; and substitute candor and generous regard, with a passion for the real truth of things in our relations in life.

It is through such organizations as the Institute and its ideals that we may look for at least a part of this harmonizing process. Institutions of learning and learned bodies in general can legitimately have as a general object such ideals, though perhaps it may without doubt require that some of them, a little hide-bound

in ancient lore, should moult at least a portion of their long-worn integument, and grow a new and more modern covering.

However all this may be, the ever-growing unity of science in all its departments, the interdependence of its advances, and the mutual aid which one department can and must give to another, is of great significance for the future.

It has appeared to me that it would be well at this time to trace briefly the origin of the Institute one hundred years ago and its activities in the century past.

THE FRANKLIN INSTITUTE.

In addressing this distinguished gathering on the subject of our Franklin Institute and the services it has rendered to scientific progress and public instruction during the past hundred years, I shall be unable to present more than a mere outline of the activities which it has sponsored; the researches which it has through so many years carried on or assisted; its influence upon education, and the outgrowth thereof—its broad fostering of the mechanic and technical arts, including its drawing and architectural schools, its assistance to governmental work; its encouragement to invention and discovery, to inventors themselves; its free library of so great value for reference; its notable and successful exhibitions; its long-established lecture courses; its valuable committee work and the many other forms of effort by which it has demonstrated its inestimable worth, not only to the immediate public in Philadelphia, but to the world at large.

When organized in 1824, chiefly through the effort and interest of Samuel Vaughn Merrick, assisted by Prof. Wm. H. Keating, of the University of Pennsylvania, the main object was to combine the idea of the Mechanics Institute, a type of society of which many examples were then being formed, with that of a learned society whose membership usually includes only those who had been selected as possessing superior knowledge in the branches to which the society was devoted.

It was recognized in those far off days that there was a great fund of practical information in possession of the skilled worker or mechanic which ordinarily was unavailable to students in the arts and sciences—and that, on the other hand, it was desirable that the more advanced knowledge of the sciences should reach those engaged in practice of the arts. Thus, the professor, the

layman, the master mechanic, the workman, could be brought together to the mutual benefit of all. This idea in a broad sense has continued through the hundred years of history of the Institute, aptly named after Franklin whose great versatility and deep interest in science discovery and the general knowledge of his time, as well as in the practice of the arts depending thereon, was so characteristic of him. We may recall that Franklin, the printer, the philosopher, the statesman, was a leading spirit in the founding here in Philadelphia of the American Philosophical Society, a learned body, the membership in which extends throughout this country and the world. That society celebrated its hundredth anniversary many years ago.

The charter of The Franklin Institute was granted by the Legislature of the State of Pennsylvania on March 23, 1824, with the title—"An Act to Incorporate The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts." Later the means whereby this promotion was to be attained were formally stated to be—

First, by the delivery of lectures on the arts and the applications of science to them; second, by the formation of a library of books relating to science and the useful arts, and the opening of a reading room; third, by the examination of all new inventions and discoveries by a committee of learned and honorable men; fourth, by the publication of a journal to contain essays on science and art, specifications of English and American patents, etc.; fifth, by holding exhibitions of American manufactures and awarding medals to worthy workmen; sixth, by building a hall for the meetings of the Institute and the use of the members; seventh, by collecting machines, minerals, materials, etc., used in the mechanic arts; eighth, by the establishment of schools in which should be taught architecture and mechanical drawing, chemistry applied to the arts, mechanics and if possible of a high school for giving young men a liberal and practical course of education.

This was indeed an ambitious program, so comprehensive in its scope that little could be thought of as desirable to add to it. It was characteristic of the men to whom, as leaders, the Institute owes its existence. It is related of Mr. S. V. Merrick that as a young man of twenty-one, he found himself "owner of a workshop, without a mechanical education, and with scarcely a mechan-

ical idea." To improve this condition, he applied for membership in a local mechanics institute and was *black-balled,* as lacking the qualifications for membership. He then apparently determined to found an organization after his own ideas.

I will not attempt to detail the steps in the formation of the society as this is already part of the records and need not be repeated here. Suffice it to say that a well-attended meeting of citizens was called on February 5, 1824, and its proceedings were eminently successful. It was presided over by James Ronaldson, Esq., who was then the leading type founder in the United States, and who continued as President of the new Institute until 1841, when Mr. Merrick, so prominent in the inception of the organization, succeeded him and held the office until 1854.

A few other names may be mentioned as among the first promotors of the new society. They are Mathias W. Baldwin, Peter A. Browne, Oram Colton, Thomas Fletcher, Robert E. Griffith, William H. Kneass, David A. Mason, James Rush, George W. Smith, M. T. Wickham and Samuel R. Wood. Many of these men were prominent citizens of Philadelphia in those early years of the nineteenth century. Philadelphia has always been a great industrial centre, with industries and arts covering the widest range, from the most delicate instruments and machines to giant locomotives and ocean liners; a range so great that it has often been said that the city had industries that could supply every need of its people with most of the luxuries added.

The Hall of the Institute on Seventh Street, above Chestnut, is now ninety-eight years old and has been made to serve the purposes of the Institute through the many years which have elapsed, but only by the most careful adaptation of means to ends. It has long since been outgrown, though its older members, like myself, will always cherish for it a sense of respect, due to the many and valuable activities of which it has been the scene.

It was in that hall that I first listened to science lectures for the public in which work, without at the time suspecting it, I was soon to take part. Becoming a member of the Institute fifty years ago (in June, 1874, in fact) I was asked to give a course of five lectures on electricity during the season of 1876–77, which I somehow had the courage to undertake, though I was then only twenty-three. I confess that it was quite an undertaking. There was incidentally, I was told, some shaking of older heads over the

choice of one so young being selected for such a serious task. I really think, looking back, that it was these very doubts that steeled me and filled me with a resolve to do my best to win success. With apparatus, much of which was new, including a small arc light dynamo—then a rarity—most of the apparatus having been constructed by myself, I set out to show that electricities of any name, static, dynamic, voltaic, electromagnetic, magneto-electric, thermo, animal or what not, were one and the same, not differing in any respect essentially. The textbooks of the day really treated of several separate electricities as if they were distinct varieties almost unrelated, except under the general title of electricity. Not to discover any vacant seats at any of the five lectures was to me a great encouragement. For several years, thereafter, I was on the list of lecturers at the Institute, becoming at last engrossed with electric developments and removing from Philadelphia to New England in 1880.

At the last of these lectures in 1877 in reversing a Ruhmkorff coil by sending a Leyden jar discharge through its fine wire secondary, so making it a primary, the first clue to what afterward became the art of electric welding was obtained. It may have been the first time that such an experiment as reversing a Ruhmkorff was ever tried.

It was the practice from the start of the Institute to lay stress on its educational objects and to this end it established schools for teaching, under control of its Board of Managers. Thus, Professor Keating, already mentioned, was appointed Professor of Chemistry; Prof. Robert M. Patterson, of Natural Philosophy and Mechanics, and William Strickland, of Architecture. There were appointed also Committees on Instruction, on Inventions, on Premiums and Exhibitions, on the Library, on Models and Minerals, etc.

The Committee on Science and the Arts, which has functioned so effectively for a long period in the past is the outcome of these early committees in at least so far as examination of new inventions and machines, together with recommendations of awards of merit, medals, premiums and the like are concerned.

The Institute had from the start several types of membership and this condition still exists, but as to eligibility for membership, there has been no restriction. The membership has been open to men and women, without discrimination as to race, nationality

or religion—all that was needed being good character and friendly interest in the objects for which the society was organized; election after proposal being left to the Board of Managers who were with the other officers elected by vote of the members in good standing.

It might at first be imagined that such democracy in the membership would not be desirable and that the objects of the new society would have been better carried out if the membership had been more restricted. The instructional value of the society was, however, so great that its meeting rooms and lecture hall became, as it were, the city's scientific and technical rostrum, for the discussion of new advances in those fields to which it had devoted itself. Always leaning toward the technical, it became a great force in the community, the activities of which were so largely given to arts and manufactures. This was and is Philadelphia. Even at the inception in 1824, the new society took steps to establish a school for the teaching of mechanical and architectural drawing, an experiment so successful from the start that the encouragement received led to the more ambitious project of providing a school in which should be taught "all the useful branches of English literature and the ancient and modern languages." Begun in 1826 the records for 1827 show that over three hundred students were enrolled. And now comes the significant thing. These Franklin Institute schools furnished a model leading to the extension of the public school system in a few years to include such a school, a People's College. In fact, it is said that the Boys' Central High School was itself patterned after the Institute schools and that the high school of the Institute was thereafter abandoned as unnecessary. The Institute retained, however, its drawing school, which has been through the many years an important part of its activities.

To me, graduated from the Boys' Central High School in 1870, and teaching in that school as assistant professor and full professor for ten years subsequently, while at the same time occupying lectureships in The Franklin Institute, serving on its Board of Managers and committees and taking part in its meetings and exhibitions, the facts just presented are naturally of unusual interest.

It is proper to recall at this time that although the Institute was organized to devote special attention to the mechanic arts,

it has at various times and in effectual ways assisted the efforts made to elevate the standards of taste in the cultivation of the fine arts as applied to the industries. The "School of Design for Women," founded by the Institute in 1850, was conducted by its committee for several years, until it became self-supporting.

The Institute also extended its friendly coöperation and sheltered during infancy the "Pennsylvania Museum and School of Industrial Art."

Mr. Frederick Fraley, well known to older Philadelphians as one who filled with great honor many positions demanding talent and integrity, and to whose unselfish devotion as treasurer, the success in considerable part of the Centennial Exhibition of 1876 was due, has recorded much of the early history of the Institute. He says: "The general interest created by the existence and working of the Institute caused more attention to be paid to technology and to science generally, and in the year 1837" (a panic year, by the way) "gave rise to a movement for the establishment of a School of Arts. The Institute headed this movement and applied to the Councils of the City for a grant of a large plot of ground in West Philadelphia as a site for the buildings of the proposed school.

"This was promptly and cheerfully granted and the legislature was appealed to by memorials from all parts of the State to endow the school by a liberal appropriation."

This project, we are told, failed of success, but was subsequently realized in the plans of the University of Pennsylvania.

That there was need of cultivation in design and in art work generally in the country was patent enough. I think this was particularly true in machine design. It was the period when steam engines, lathes, planers, and the like were built on architectural designs, including moulded cornices, fluted pillars and the rest. Locomotives were painted in all the bright colors, scarlet, chrome yellow, grass-green, as agricultural machinery was later and some of it is to-day.

Besides its educational influence and activities, a most important undertaking of the Institute was the establishment of its library, which in time became of a distinctive character in our country as a scientific reference library, exceptional in its accessibility and completeness. It contains a great collection of the publications of principal scientific and technical bodies here and

abroad, as well as the leading periodicals dealing with science and the useful arts. It also became in 1887 a public depository for the congressional district in which it is located. In this way, it was the custodian of valuable publications which rendered much assistance later in the completion of sets of the publications of the various scientific and technical bureaus of the government.

As a patent reference library, including United States and foreign patents, the Institute library has been most valuable to inventors and others desirous of finding readily the status of ideas or inventions, or to those wishing to obtain a review of an art with the object of its improvement. So valuable a library as that of the Institute badly needs space for its growth, overcrowded as it long has been. Ample protection from fire is indeed a necessity, long ago recognized as most desirable.

As a member of the Institute I was fifty years ago a frequent visitor to this library, which was of great value in my early studies in the sciences and their application. Its very atmosphere and the method of its conduct were alike a stimulus to one in search of knowledge. This was many years ago, but I have no reason to believe that any of this has changed in the lapse of time during which its collections have been much enriched.

I cannot leave this subject of the library without expressing the hope that it may soon be securely housed in new Institute buildings long contemplated to be built and that ample measures may then be taken to facilitate the use of so valuable a collection of the literature in science and technology.

Not in small measure indeed has the Institute itself contributed to this fund of science literature. It has in the publication of its journal, the JOURNAL OF THE FRANKLIN INSTITUTE, extended its usefulness far beyond the confines of the city and state which is its domicile. Appearing originally in 1826, under the title, *The Franklin Journal and Mechanics Magazine,* it was published under the patronage of the Institute, and its editor was Dr. Thomas P. Jones, Professor of Mechanics in the Institute. With the January issue of 1828, however, it was named the JOURNAL OF THE FRANKLIN INSTITUTE, which name has been retained.

In the Franklin JOURNAL, it was originally intended to publish a list of patented inventions with remarks on their utility and originality. In fact, this idea was definitely carried out and remained a specially prominent feature of the JOURNAL up to the

close of 1859, with the exception that the remarks on utility and originality were discontinued, when in 1848, Doctor Jones, who had continued as editor from the start, died. He had in the meantime been made superintendent of the U. S. Patent Office, which fact doubtless led him to give attention to the records of patents as part of the text of the JOURNAL itself.

It happened that in the Government Patent Office publication, prior to 1843, there was no printed account of the claims of the patents as issued and these could be found only in the Institute JOURNAL, which for the period of 1826 to 1859 had given the claims as well as abstracts of the specifications of the patents issued by the United States. Thus, the JOURNAL became and remained the only source available at any time for a record of the claims of patents issued between 1828 and 1842, inclusive.

This circumstance throws a curious side light on the comparative crudity and incompleteness of the patent system in the United States even eighty years ago and within the early years of the formation of the Institute. It is probable that we have now in the United States a patent system which has no superior anywhere. To its liberal patent system with its encouragement of invention, we may perhaps trace in major part the reputation for inventiveness acquired by our people. To such a system, facility in determining what is new and what is old, is second cousin. The library of The Franklin Institute, replete as always with patent publications, has provided just such facility. It has also brought to the earnest student access to the scientific and technical journals and other publications in a measure not easy to be overestimated.

All this valuable work has been done with a relatively very limited endowment and with an equally limited library force. It is a free public reference library, containing an exceptional collection of books, pamphlets, journals, maps, charts, photographs, etc., bearing on science and its applications.

I think that it can be said truly that the JOURNAL of the Institute has steadily improved with the flight of years, so that the papers published therein, largely based on the lectures and discourses at the Institute itself, have reflected the general advance in science, in its latest and most recondite aspects. It has achieved a most enviable reputation, both here and abroad, as to the standing and general excellence of the material which appears in its pages.

This necessarily brief account of the objects and services of The Franklin Institute would be very imperfect if some allusion were not made to its function of investigation, examination and award accomplished through its committees during the century of its life.

Early in its history, a special committee of the Institute made a careful investigation of water-wheel efficiency, examining the various types then known, and the results appeared in the JOURNAL. This was followed by an investigation by a committee of the cause of steam-boiler explosions, not very uncommon in those early days. In this investigation, the committee secured the coöperation of the government, with an appropriation therefrom to defray the cost of the experiments, without, however, any compensation to the members conducting them. It was volunteer work through many months and resulted in great public benefit. Allied to these boiler tests was an examination of the strength of materials used in construction, the results also being published fully in the JOURNAL.

A notable page in the history of the Institute's activity and service is the interest elicited in the study of meteorology by lectures and essays of Prof. James P. Espy, leading to the appropriation by the State of $4000 in 1843 for the acquisition of instruments and establishment of observing stations throughout the State of Pennsylvania. The Committee of Meteorology of the Institute, in conjunction with a like committee of the American Philosophical Society, conducted the observations for the study of weather conditions and storms.

This early meteorological work gave rise to a theory of storms which foreshadowed in large measure the views subsequently adopted. It was a work carried on before the days of telegraphy and was later followed by the establishment of weather bureaus in this and other countries. There is no need to speak of their value in agriculture and in commercial enterprises. In 1887, following the lead of The Franklin Institute, there was begun the Pennsylvania State Weather Service, which has been so successful in later years.

To mechanical engineers and machine manufacturers, the results of the work of the special committee for studying the shapes and proportions of screw threads is well known. As far back as 1865, its report was adopted by the Institute. It proposed

for adoption a simplified system of screw threads which shortly thereafter was adopted by the government and called "The United States or Franklin Institute Standard Thread"—adopted throughout the country. The records show that the same system, modified only in dimensions to suit the metric system, was reported favorably by The Associated Engineering Societies of Germany in 1887.

In 1875, an expert body nominated by the Institute coöperated with the Philadelphia Water Department in considering the then present and the future water supply of the city, making an elaborate report, published in abstract in the JOURNAL at the time.

Reference should also be made to the investigation of the 1877–78 Committee on Dynamo Electric Machines, then rare machines, used in operating single arc lights. It was probably the earliest inquiry of the kind and served to bring out much important data, as well as to reveal certain characteristics before unknown in relation to the electric arc itself, and to the relation of resistance internal and external of the circuits, voltage, etc., as bearing on the efficiency in the utilization of power applied to drive such machines.

It was for the first time shown that the greater the external or work resistance was in relation to the internal resistance of a dynamo, the higher the efficiency, other things being equal. I had some years before this been constructing small dynamos and studying them. They were indeed rare machines in those days. Serving as I did on the Committee as one in charge of electrical determinations, the work was to me most inspiring and valuable as a foundation for my future efforts in the field.

The work of the Committee preceded the appearance of the carbon filament lamp of Edison by nearly two years. The report of the Committee is to be found in May and June numbers of the JOURNAL of 1878.

About six years afterward, at the time of the Institute's International Electrical Exhibition in the fall of 1884, the first electrical exhibition held in America, the earlier work in the electrical field was supplemented by more elaborate reports of similar scope, but the great growth of the electric lighting industry, which had taken place in the intervening years, furnished an abundance of material which only emphasized the paucity of that to which the early work was restricted. A notable feature of the work at the

exhibition in 1884 was the report on the "Life, Duration and Efficiency of Incandescent Lamps."

The Institute published in its JOURNAL of December, 1881, a report on "The Conditions of Safety in Electric Lighting." It is probably true that it was a first exposition of a number of conditions to be followed in wiring of buildings and installation of circuits, afterward incorporated in Underwriters' rules.

In the foregoing brief outline of the long work past of some of its committees, an account which could, if there was need, be greatly extended, no mention has been made of the practically perpetual Committee on Science and the Arts. Continued in one or other form from the very start of the Institute, this Committee has up to the present been an important factor in carrying out the original objects of the founders in 1824. At first it was a standing committee of five members called a Board of Examiners, the duty of which was to examine and report on new and useful machines, inventions and discoveries submitted to them. Later the name was changed to "Committee on Inventions." Continuing in this form and enlisting the voluntary services of members notable for their attainments in mechanics, it was replaced in 1834 by the "Committee on Science and the Arts," and at the same time its scope was enlarged. The records disclose a rather ambitious set of functions, continued for more than fifty years, the JOURNAL being much enriched by the labors of the body. This committee had been, prior to 1886, a voluntary association of members, but was reformed in that year on the elective basis, consisting of forty-five members, fifteen being elected each year. The condition of their choice was that they "shall pledge themselves"—"to perform such duties as may devolve upon them, and to sustain by their labors the scientific character of the Institute."

An important duty of the Committee, calling for care and discretion, is in the awards of medals, certificates of merit and diplomas. The Elliott Cresson gold medal, named after its founder, is awarded as a mark of recognition for inventions and discoveries of unusual or permanent value. The Committee is empowered, according to its judgment, to award "The Medal of Merit" (made possible by the bequest of Edward Longstreth, at one time Vice-president of the Institute), also The Franklin Medal, founded by Samuel Insull. "The John Scott Legacy

Medal and Premium" was awarded on the recommendation of the Committee on Science and the Arts from 1835 to 1918, a period of over eighty years, the action of the Committee having been followed in making this award by the Board of City Trusts. It was established by John Scott, of Edinburgh, Scotland, as a recompense to ingenious men and women who make useful inventions. This medal is not now awarded as formerly on the recommendation of this committee.

Ten years ago there was established The Franklin Medal by Samuel Insull, president of the Commonwealth Edison Company of Chicago, and the award of this medal is now esteemed the highest honor which the Institute has in its power to bestow. The first Franklin Medals were awarded in 1915, one to Thomas A. Edison and another to Heike Kamerlingh Onnes, the great leader in low temperature research.

The fact that throughout a century such work has been carried on with so satisfactory results and with unwavering honesty of purpose, is in itself a testimonial to the fidelity with which it has performed the tasks set for it, while the services of its members have at all times been rendered without compensation. It is most unlikely that such a record can be matched anywhere. All the members of the Committee have been chosen by the Institute, as far as possible, because of special competency as experts in professions or trades, eminent in their attainments and reputation. This prestige has been of the greatest value to the standing of the Institute at large before the public.

EXHIBITIONS.

In the early conception of the organization of our Institute, the holding of exhibitions for showing to the public the products of industries and the advance in the mechanic arts and technical science was made an important aim. It was thought that thereby encouragement might be given, and stimulation of a healthy rivalry among the exhibitors secured, which would result in lasting benefit to them and to the public. This was at a period when American manufacturers were in their infancy and needed encouragement to enable them to effectually compete with foreign productions. The early exhibitions appear to have been successful, and the records of them disclose interesting facts concerning the inception and growth of manufacturing in the United States.

Premiums were awarded for meritorious accomplishment. To this end and to gain the confidence of the exhibitors, skilled examiners or judges were chosen for weighing the merits of the exhibits themselves.

There can be no question that the conduct of these exhibitions served to increase the influence of the Institute very greatly, and to inspire confidence (in the public mind), as well as to stimulate a sense of loyalty and pride in the members.

These exhibitions were held either yearly or biennially up to the year 1858, and in various places available at the time.

Naturally the reports and awards of premiums and medals were much prized by the recipients, while the exhibitions themselves served to nurture and encourage young and growing industries and improve those already firmly established. The historian finds a rich mine of information in the records of the exhibitions in the accumulation of data concerning the origin and progress of the industrial arts, thus brought together.

However, the two most notable exhibitions held by the Institute were those of 1874 and 1884, the former commemorating the fiftieth anniversary of the founding of the society, and that of 1884 being distinctive in that it was the first international electrical exhibition held in America. It may be said to have commemorated the establishment of the early electric industries soon after to become of the greatest importance not only to progress in the arts themselves, but in the development of our social system. As I have pointed out before this, the great structures such as the tall and multifariously cellular buildings called skyscrapers, characteristic particularly of New York City, could not exist without three electric agencies: The electric light (gas would have made furnaces of them); the telephone, the absence of which would have been fatal in itself, and the electric elevators, the high-speed vertical railways for quick passenger traffic between floors from top to street and even below street level.

I remember well the 1874 exhibition, held as it was in the old Pennsylvania Railroad Station at Thirteenth and Market Streets, where the Wanamaker stores, well known to every Philadelphian, were erected soon after. It was in every sense a well-planned industrial exhibition, worthy of the Institute, and served much to increase the prestige of the society. I had myself been

elected to membership in June of 1874, and my first duty as a member was to serve on a committee of judges of exhibits in the field of scientific apparatus, optical and electrical chiefly. It was my first experience of the kind and being but twenty-one years old at the time, the sense of responsibility engendered is a lasting remembrance.

It was at this exhibition that the Tilghman sand blast process was shown as a novel development, which in later years became of signal importance in several arts. I cannot remember that a single dynamo generator of electricity formed part of this exhibition, excellent as the showing was in many departments of manufacture, machinery and the inventions of the time.

Yet, in less than ten years from 1874, there had come into existence systems of arc and incandescent electric lighting, destined to revolutionize the whole art of illumination.

At the Centennial Exhibition of 1876, there were, as I remember, only two types of dynamos displayed, the Gramme and Wallace-Farmer, the latter of which soon disappeared after The Franklin Institute committee's tests of 1877–78 had shown its very low efficiency as compared with the Gramme dynamo, and after the causes of that inefficiency had been pointed out by the committee. In less than two years, or in 1879, there appeared the series arc lighting systems of Brush of Cleveland, the Thomson-Houston series system, with its automatic regulation, in Philadelphia, the Weston system in Newark, New Jersey, and others; and these several systems were working that year in a number of places for lighting of spaces to which the arc lamp was best adapted. The principles of dynamo construction and operation began to be fairly well understood by those who were connected with their design.

At the very close of 1879, Edison announced the incandescent lamp with its carbon filament, sealed bulb and high vacuum, and made the first showing of a few such lamps at Menlo Park, New Jersey. Central stations for arc lamp distribution in cities were beginning to be established and a year or two later, the Pearl Street Station, the first for the distribution of incandescents on the Edison plan, was established in New York. This was followed a year later by the improvement known as three-wire distribution which was first installed in a lighting station in the city of

Brockton, Massachusetts, in 1883. This brings us close to the important Institute exhibition, entirely electrical in character and international in scope, which was opened in the fall of 1884. This exhibition was not only an excellent display of the latest developments in the larger electrical applications, but it served to bring together the scientific men here and abroad in a notable congress, the meetings and discussions held adding greatly to the knowledge of principles of the new industry. My own first meeting with Sir William Thomson, afterward Lord Kelvin, the centenary of whose birth was celebrated in London in July last, occurred at these meetings in 1884. Along with Sir William Thomson were scientific men of international reputation such as Lord Rayleigh, Prof. Silvanus P. Thompson, William H. Preece, Prof. George Forbes and others. Professor Forbes I had the pleasure of meeting in London lately—and he is the only survivor of those named. Of our own country, there may be mentioned such names as Prof. Henry A. Rowland, Prof. George F. Barker, Simon Newcomb, Prof. C. F. Brackett of Princeton, Prof. F. Willard Gibbs, Prof. John Trowbridge, Prof. Charles A. Young and others of high standing in science at that time. None of these now survives.

The several international electrical congresses, which have generally been held in connection with the larger International Exhibitions, have had for their principal end the establishment of uniform definitions, designations and values for the fundamental units, such as the ohm, the volt, the ampere, the henry, etc., and it is owing to these bodies that not only the names, but the values and methods of determining such values have become uniform throughout the world. At the International Electrical Exhibition of 1884, the Electrical Congress was the first held in America, followed later by the important gathering of International Delegates at the World's Fair in Chicago in 1893 and still later by the Chamber of Official Delegates at St. Louis in 1904. Following this international gathering, there was established the International Electrotechnical Commission, the work of which has continued uninterruptedly to the present time.

The educational value of the exhibitions has unquestionably been very great, yet such has been the advance in technology since the beginning of the present century and before that, that an

exhibition, even restricted in its scope, would at once become too vast and beyond the power of such a body as the Institute to properly finance and organize. Hence, exhibitions of the character of those of 1874 and 1884, to be at all adequate in representation, must necessarily, it seems, be held in the future, if at all, under the auspices of governments which also furnish the financial support as in constructing suitable buildings, installing exhibits and providing for the many activities which are necessarily involved.

The British Empire Exhibition now being held in London is an example of the type which must survive in the future.

These facts cannot, however, dim the fame of the Institute as to the successful conduct of the exhibitions which have characterized its past. Abundantly useful as the Institute has been in the past century in furthering scientific knowledge, particularly as applied to the arts; and numerous as have been the instances of brilliant work accomplished, can we doubt that, under proper guidance and with the firm policies which have become characteristic of it, the future holds equal promise of accomplishment. It is for the public to realize the true value of the possible services it can render and accord proper support. Endowments such as the Bartol Research Foundation are needed and will be more needed in the future than in the past. The ramifications of applied science naturally increase, and the delicacy and cost of research grows, the deeper we penetrate Nature's secrets. There must be an increasing corps of workers, with the growth of extent and variety of problems presented for solution. It is hoped that the Institute may receive generous support in the coming years and that its usefulness in carrying out the original objects for which it was founded may not be restricted by inadequate means or provision. Those among us of the older generation can hardly hope to see much of its growth, but the heritage which the younger generation has received from the past should be the stimulus to continue and increase the good work which so far has characterized the work of the Institute in its various forms. While the associations which have grown up around the old Hall on Seventh Street stir many emotions in many of the older members, yet it is to be assumed that not many more years will elapse before the proposed new hall and library on the new site may become a reality.

THE FIFTH ESTATE.*

BY

ARTHUR D. LITTLE, Ch.D.

Member of the Institute.

BENJAMIN FRANKLIN was not perhaps in all respects a paragon, but he was unquestionably a polygon—a plain figure with many sides and angles. There were not enough buttons on his black coat to tell off the multifarious aspects in which his complex personality was presented to the world. He was craftsman and tradesman; philosopher and publicist; diplomat, statesman, and patriot. And he was, withal, a very human being. What concerns us particularly on this occasion is the fact that he was at once philosopher and man of affairs. His remarkable career should refute forever the fallacy, which, unfortunately, still is current, that the man of science is temperamentally unfitted for the practical business of life.

At the time when Franklin was in England the British Parliament was assumed to be composed of representatives of three estates: The lords spiritual, the lords temporal, and the commons, but Edmund Burke, pointing to the Reporters' Gallery, said, "There sits a *Fourth Estate,* more important far than they all." No one at all familiar with the ubiquitous influence and all-pervading power of the Press would to-day question the validity of Burke's appraisal. Even then, however, there was present in England, in the person of Benjamin Franklin, a prototype and exemplar of the membership of a *Fifth Estate,* an estate destined to play an even greater part than its predecessors in the remaking of the world.

This Fifth Estate, to which your attention is appropriately invited on the centenary of The Franklin Institute, is composed of those having the simplicity to wonder, the ability to question, the power to generalize, the capacity to apply. It is, in short, the company of thinkers, workers, expounders, and practitioners upon which the world is absolutely dependent for the preservation

* Address delivered in the hall of the Academy of Natural Sciences, Friday, September 19, 1924, on the occasion of the observance of the centenary of the founding of The Franklin Institute.

and advancement of that organized knowledge which we call "Science." It is their seeing eye that discloses, as Carlyle said, "the inner harmony of things; what Nature meant." It is they who bring the power and the fruits of knowledge to the multitude who are content to go through life without thinking and without questioning, who accept fire and the hatching of an egg, the attraction of a feather by a bit of amber, and the stars in their courses as a fish accepts the ocean.

The curious deterioration to which words are subject has left us with no term in good repute and common usage by which the members of the Fifth Estate may properly be characterized. Sophists are no longer distinguished for wisdom; they are now fallacious reasoners. Philosophers, who once claimed all knowledge for their province, are now content with speculative metaphysics. Scholars have become pupils. The absent-minded and myopic professor is a standardized property of the stage and screen. The expert, if not under a cloud, is at least standing in the shade. In Boston, one hesitates to call a professional man a scientist, he may be a Presbyterian; and a "sage," as an anonymous writer has pointed out, "calls up in the average mind the picture of something gray and pedantic, if not green and aromatic." Let us, therefore, for a time at least, escape these derogations and identify ourselves as members of the Fifth Estate.

Although the brotherhood of the Estate is open to all the world, its effective membership nowhere comprises more than an insignificant proportion of the population. Two hundred and fifty constitute the membership of the National Academy of Sciences. The latest edition of "American Men of Science" includes only about 9500 names. The number is expanded to 12,000 on the roll of the American Association for the Advancement of Science. Although gathered from all countries and though chemistry is one of the most active and inclusive of the sciences, the chemical papers, books, and patents reviewed in *Chemical Abstracts* in 1923 were the product of about 22,000 workers. One may hazard the estimate that there are not in all the world 100,000 persons whose creative effort is responsible for the advancement of science.

The studies of Cattell indicate that in America, at least, the great majority of men of science come from the so-called middle and upper classes, or precisely those sections of society which, in

Russia, have been practically exterminated in the name of the new Social Justice. In about two-thirds of Cattell's reported cases both parents were American-born, while the fathers of nearly one-half were themselves professional men. Seventy-five per cent. are dependent upon the universities for support, from which we may assume that the burden of the higher surtaxes does not bear heavily upon the Fifth Estate.

In proportion to population the cities have produced twice as many scientific men as the country, but how many "hearts once pregnant with celestial fire" repose in country churchyards because of lack of opportunity and absence of the stimulus of contact, cannot, of course, be known; nor can we tell how many brains, competent and equipped to penetrate the mysteries of nature, the war has cost the world.

Initiative is one of the rarest mental qualities; yet without it progress is impossible. Its combination with the scientific imagination and command of fact is still rarer and more precious. Since comparatively few of those who study science develop the capacity to extend its borders the cost of a man competent to advance science has been estimated at $500,000 and his value to the community set at a far greater figure. Full membership in the Fifth Estate thus seems to involve the highest initiation fee on record. It is a figure disconcerting to the candidate, but as Wiggam has finely said: "Only genius can create science, but the humblest man can be taught its spirit. He can learn to face truth."

That the Fifth Estate is not better appreciated or always understood by the world at large is not surprising. In their endeavors to secure accuracy of definition and expression its members have evolved a preposterous and terrifying language of their own. It is not ideally adapted to the interchange of confidences in ordinary human intercourse. It does not lend itself to poetry. "Ladybird, ladybird, fly away home" becomes impossible when one is forced to address the prettily spotted beetle as *Coccinella dipunctata*. A primrose by the river's brim is much more than a yellow primrose to the botanist: it is a specimen of *Primula vulgaris*. The organic chemist produces a new synthetic product in a mass of pilular dimensions and bestows upon it a name that would slow up Arcturus. Nothing but static interfer-

ence can account for the terms of radio-telephony. If knowledge is to be humanized it must first be translated.

Dewar has said that the chief object of the training of a chemist is to produce an attitude of mind. It should be the object of all education to produce the scientific attitude toward truth. We may even agree with Robinson that "of all human ambitions an open mind, eagerly expectant of new discoveries and ready to remould conviction in the light of added knowledge and dispelled ignorances and misapprehensions, is the noblest, the rarest and the most difficult to achieve."

Carlyle says, "The degree of vision that dwells in a man is a correct measure of the man." And President Coolidge has been quoted as saying in a recent interview: "Everything flows from the application of trained intelligence, and invested capital is the result of brains. . . . The man of trained intelligence is a public asset. . . . We go forward only through the trained intelligence of individuals but we, not the individuals, are the beneficiaries of that trained intelligence. In the very nature of things we cannot all have the training but we can all have the benefits."

Now vision, a trained intelligence, and an open mind are the qualities which characterize all those who are worthy of membership in the Fifth Estate. They are qualities which the many-sided Franklin possessed in exceptionally high degree.

Among all the activities with which his busy life was crowded Franklin undoubtedly found his greatest pleasure in the pursuit of science, and in that pursuit he followed the eclectic method. At a time when nearly everything awaited explanation his focussed attention ranged like a searchlight over many fields. He observed the movement of winds and developed a theory of storms. He considered ventilation and the causes of smoky chimneys and proceeded to invent new stoves. He introduced the Gulf Stream to Falmouth skippers and demonstrated the calming effect of oil on turbulent seas to officers of the British Navy at Portsmouth. From earthquakes he turned to the heat absorption of colored cloths and the fertilizing properties of gypsum. He wrote on sun spots and meteors; waterspouts, tides, and sound. The kite, which for centuries had been the toy of boys, became in Franklin's hands a scientific instrument, the means to a great discovery. That its significance is, even now, not universally appreciated is shown by the recent answer of a schoolboy, "Lightning differs from elec-

tricity because you don't have to pay for lightning." To Franklin, as the child of Everyman knows, we owe our initial conceptions of positive and negative electricity, and he was the first to suggest that the aurora is an electrical phenomenon.

The gregariousness, which is a prominent characteristic of the Fifth Estate, found early expression in Franklin. He formed The Junta, a club for the discussion of morals, politics, and natural philosophy, and in 1744 drew up a proposal for the organization of the American Philosophical Society, of which he later became President. He established a wide acquaintance and cemented many firm friendships among the foremost scientific men of France and England, by whom he was received on equal terms. In 1753 he was awarded the Copley medal of the Royal Society for his discoveries in electricity, and on his leaving England, David Hume wrote: "I am sorry that you intend soon to leave our hemisphere. America has sent us many good things, gold, silver, sugar, tobacco, indigo, but you are the first philosopher and indeed the first great man of letters for whom we are beholden to her."

The professional spirit which animates the Fifth Estate is essentially one of service. Its compelling urge in the search for truth springs from the conviction that the Truth shall make men free. That spirit finds complete expression in Franklin's statement: "I have no private interest in the reception of my inventions by the world, having never made, nor proposed to make, the least profit by any of them." This impersonal relation to the children of his brain was indeed carried by him to an extent which ordinary human nature would find hard to emulate. "I have," he writes, "never entered into any controversy in support of my philosophical opinions; I leave them to take their chance in the world. If they are right, truth and experience will support them; if wrong, they ought to be refuted and rejected."

There is, nevertheless, a place for militancy in science. The world needs a Huxley for every Bryan.

Franklin was a man of science, but his career proclaims that it is possible to be a man of science and much more besides. Science was made for life, and life is more than science. Art in its fullest expression may touch deeper springs, human relations and affections may bring richer rewards, and public affairs may make a more imperious claim. With Franklin as their prototype,

the members of the Fifth Estate may well strive to emulate his devotion to the public service and his broad and constructive interest in human problems and affairs.

Error and misconception have a feline tenacity upon life, and the Fifth Estate, though richly endowed with latent executive capacity, is still in popular opinion regarded as equipped for thought rather than for action. The practical man, busily engaged in repeating the errors of his forefathers, has little time and less consideration for the distracting theories and disconcerting facts of the man of science. Yet who, among the men of action, is more intensely and truly practical than Carty, Baekeland, Reese, or Whitaker? Where shall one find a firmer grasp on the details of business than that possessed by E. W. Rice, Jr., Gerard Swope, or Doctor Nichols? What quality caused the young director of a research laboratory to find himself responsible for the production of gas masks to protect four million fighting men? In a time of dire emergency it was a professor of chemistry who organized the great Edgewood Arsenal and developed the means and methods and the trained personnel required to supply munitions for a new type of warfare. It was not to a statesman or a business man or a great manufacturer that the Allies entrusted the supreme command. It was to a teacher in a French military school. The range and value of their public service obscures the fact that Charles W. Eliot was a professor of chemistry and that Herbert Hoover is an engineer. The League of Nations is the child of a schoolmaster.

Numerically the Fifth Estate has always been feeble and insignificant. Its total membership at any time could be housed comfortably in a third-rate city. No politician makes a promise or invents a phrase to attract its scattered and ineffective vote. Rarely do its members sit in Congress: when they do, they sit in the gallery.

With less political influence than the sparse population of Nevada, the Fifth Estate has recast civilization through its study and application of "the great and fundamental facts of Nature and the laws of her operation." It has opened out the heavens to depths beyond imagination, weighed remote suns and analyzed them by light which left them before the dawn of history. It has moved the earth from the centre of the universe to its proper place within the cosmos. It has extended the horizon of the mind

until its sweep includes the 30,000 suns within the wisp of smoke in the constellation Hercules and the electrons in their orbits within the atom. It has read the sermons in the rocks, revealed man's place in nature, disclosed the stupendous complexity of simple things, and hinted at the underlying unity of all.

Because of this new breadth of vision, this lifting of the corner of the veil, this new insight into the hidden meaning of the things about him, the mind of man, cramped for ages by taboos and bound by superstition, is emerging into freedom: into a new world, rich in promise, and of surpassing interest and wonder.

Man brought nothing into the world and through long and painful ages he added little to that nothing: a club, an axe of stone, a pebble in a sling, some skins of beasts, a rubbing of sticks for a fire. He might labor, but to what avail? Even to-day the South American Indian works incessantly, yet his labor produces little more than heaps of stones. To those who would have us believe that all wealth is produced by labor the Fifth Estate replies, "Wealth is the product of brains, and labor is productive only as it is guided by intelligence."

Science is the great emancipator of labor. Bagehot has somewhere said, perhaps in "Physics and Politics," that, during the early stages of civilization, slavery was essential to progress because only through the enforced labor of the many could the few have leisure to think. To-day, in the United States, the supply of available energy is equivalent to sixty man-power for every man, woman, and child. There is now leisure for all to think, but the millions prefer the movies.

It is not labor, but the trained intelligence of the Fifth Estate which has endowed man with his present control of stupendous forces. It has solved problems that for ages have hindered and beset mankind. It has revealed great stores of raw materials, synthesized scores of thousands of new compounds, furnished the fundamental data which find embodiment in machines and processes and in those agencies of transportation and communication that have made of the world a neighborhood. It has enabled man effectively to combat disease, added years to the average life, and made it better worth the living.

Benjamin Franklin died in 1790—one hundred and thirty-four years ago. Could he return to make appraisal, what wonders

would confront his astonished vision, what triumphs of the Fifth Estate compel his admiration!

Electricity, which to his contemporaries was little more than an obscure force, the curious manifestations of which might supply an evening's entertainment, has become the structural basis of the universe. The atom of Democritus is now a microcosm, vibrant with energy that glows in the white light of the electric lamps, which have replaced the tallow dip. In place of the electrophorus and the charges of the Leyden jar he would find in our own country alone twenty-seven million horsepower driving generators in thousands of stations from which electric energy is distributed to our homes and factories and transportation lines to perform innumerable services. Imagine, if you can, the stunning impact of the impressions that would crowd the day of his return. With what amazement would he converse over a wire from Philadelphia to San Francisco or hear a voice transmitted through the ether from a point halfway around the world. So commonplace a thing as a street car would leave him open-mouthed with wonder, which might well increase at sight of an electric locomotive, hauling its thousands of tons of freight.

In great industrial plants he would find electricity driving machines of an intricacy, precision, and productive power beyond the imagination of his generation, or at work in decomposing cells and in the heart of glowing furnaces fashioning new products. In university and corporation laboratories would be revealed to him the marvels of the X-rays, photography, the fascinating world of the microscope, balances weighing 1/100,000th of a milligram, the spectroscope, and all those instruments of precision and research which are the tools of the Fifth Estate. Elements unknown to him would be placed in his hand; fascinating experiments performed to demonstrate properties and relationships beyond his dream. The air, which he studied with reference to winds, combustion, and ventilation, would be reduced before him to a liquid as obvious as water, though boiling on a cake of ice.

Where once the postboy and the post chaise were familiar, he would find our roads crowded with automotive vehicles and the country gridironed by the railways. Should he wish to send a letter across the continent, he would have only to commit it to the air mail to ensure its arrival in thirty-six hours. Were he called upon to revisit England, there would be no ten weeks' voyage

in a sailing packet, but the speed and luxury of a 50,000-ton liner, oil fired and turbine driven. At Portsmouth, where he calmed the waves with oil, he would find, instead of wooden frigates and smooth-bore cannon, submarines and armored superdreadnaughts, a single gun of which could sink the entire British Navy as he knew it. Did he wish to proceed to Paris? He would have only to take passage in an aeroplane.

The gardeners Franklin knew grew peas for pleasure or profit. Mendel grew them and established the laws of heredity. Farming, which was a wholly empirical occupation, is now the special concern of a great governmental department devoted to the development of scientific agriculture. Here Franklin would learn of soil analysis and seed selection, of hardier and more prolific varieties of plants, of better breeds of animals, of methods of control of such virulent diseases as splenic fever, anthrax, hog cholera, and bovine tuberculosis. He would find his own experiments with gypsum extended to cover the whole field of chemical fertilizers, the air itself converted into an inexhaustible reservoir of plant food, and the efficiency of farm labor multiplied many times by ingenious agricultural machines.

He would find household economies revolutionized: the town pump replaced by running water; electricity a servant in the house; the food supply broadened and stabilized; domestic drudgery assumed by laundry, bakery, and factory; tasteful clothing within the reach of all; transportation and amusement for the multitude, and the history of yesterday sold for a penny. Innumerable new industries, based on the findings of the laboratory, now offer the means of decent livelihood to millions and open careers to thousands.

In great hospitals, permeated with the scientific spirit and equipped with many new and strange devices for the alleviation of human suffering, he would hear of the incalculable benefits which medical and surgical science have conferred upon mankind. He would see the portraits and listen to the story of Pasteur and Lister and Loeb and Ehrlich. We know to-day with what joy and relief the world would welcome a veritable cure for cancer, but we can little realize the emotion with which one like Franklin would learn in a single afternoon of the germ theory of disease, of preventive serums, of antisepsis, of chemotherapy, of the marvellous complexity of the blood stream, and of the extraordinary

influence and potency of the secretions of the ductless glands. What appraisal would he make of the service to humanity which, in little more than a generation, has mitigated the horrors of surgery by the blessings of anæsthesia and antisepsis, which has controlled rabies, yellow fever, typhoid fever, tetanus; which is stamping out tuberculosis, curing leprosy, and providing specifics for other scourges of the race. What values would he put on insulin, thyroxin, adrenalin? The physician is no longer compelled to rely on herbs and simples and drastic mineral compounds of doubtful value and uncertain action. Compounds of extraordinary potency, isolated or synthesized by the chemist, are now available to allay pain, correct disorders, prolong life, and even to restore mentality and character.

With contributions to their credit, which have so enriched and stimulated the intellectual life; which have brought the peoples of the earth together into closer touch than English shires once were; which have revolutionized industry, enlarged the opportunity of the average man, and added so greatly to his comfort and well-being, we may reasonably inquire, "What are the recompenses of the Fifth Estate?"

On the material side they have almost invariably been curiously inadequate and meagre. It is incomparably more profitable to draw The Gumps for a comic supplement than to write "The Origin of Species." There is more money in chewing gum than in relativity. Lobsters and limousines are acquired far more rapidly by the skilful thrower of custard pies in a moving-picture studio than by the no less skilful demonstrator of the projection of electrons. The gate receipts of an international prize fight would support a university faculty for a year.

One may recall that Lavoisier was guillotined by a republic that "had no need of chemists;" that Priestley was driven from his sacked and devastated home; that LeBlanc, after giving the world cheap alkali, died in a French poorhouse; that Langley was crushed by ridicule and chagrin in his last days. A month before the war who could have believed that within a few years the Fifth Estate in Russia would be utterly destroyed and in Germany and Austria existing at the very edge of starvation. What has happened there may happen again elsewhere if the intelligence of the world does not assume and hold its proper place in the direction of national and world affairs.

In the preface to his recent "Lehrbuch der Photochemie" Professor Plotnikow has written: "Home and property were pillaged by bands of idle Russians who used my library for cigarette papers. Hunger, misery, want, and personal insecurity, often approaching fear for my life, were the constant accompaniment of my labors."

One is reminded that Carlyle, on the authority of Richter, says: "In the Island of Sumatra there is a kind of 'Light-Chafers,' large Fire-flies, which people stick upon spits, and illuminate the ways with at night. Persons of condition can thus travel with a pleasant radiance, which they much admire. Great honour to the Fire-flies. But——!"

It is not becoming that the world expect the light to shine indefinitely when carrying a lantern is often less remunerative than carrying a hod. The money and the years of study required for special training are not recognized as invested capital, and the return from a decade of research is often taxed as the income of a year. Professorial salaries move forward as slowly as a glacier, but they seldom leave a terminal moraine. Yet teaching is our most important business, for a failure to pass on for a single generation the painfully accumulated knowledge of the race would return the world to barbarism.

Though material wealth is rarely acquired by the Fifth Estate, its members have the riches of the royal man, defined by Emerson as "he who knows what sweets and virtues are in the ground, the waters, the plants, the heavens, and how to come at these enchantments." Their wealth is in the Kingdom of the Mind. It is inalienable and tax-exempt. It may be shared and yet retained.

A recent survey by a national magazine would seem to indicate that the majority of men have drifted into their vocations with little effort of selection and that a very large proportion ultimately regret their choice. This is seldom true of members of the Fifth Estate. Theirs is a true vocation, a calling and election. It brings intellectual satisfactions more precious than fine gold. They live in a world where common things assume a beauty and a meaning veiled from other eyes; a world where revelation follows skilful questioning and where wonder grows with knowledge. Together they share the interests, the communion of spirit, the labors and the triumphs of the fraternity of science. The Law of Diminishing Returns exerts a control from which there

is no escape in agriculture, industry, and business. Research alone is beyond the twelve-mile limit of its inhibitions.

If " the Heavens declare the glory of God " that glory is surely made more manifest by telescope and spectroscope. If the whirling nebulæ and the stars in their courses reveal omnipotence, so do the electrons in their orbits reveal His presence in universes brought into being by the striking of a match. The laboratory may be a temple as truly as the church. The laws of nature are the will of God, their discovery is a revelation as valid as that of Sinai, and by their observance only can man hope to come into harmony with the universe and with himself.

There has been a general and ready acceptance by the world of the material benefits of science, while its contributions to sociology and ethics are as generally ignored as guides to human conduct. Yet science proclaims new commandments as inflexible as those engraved on stone and furnishes what Wiggam has reverently termed " the true technology of the Will of God."

Science has so drawn the world together and so rapidly remoulded civilization that the social structure is now strained at many points. Statecraft and politics, law and custom lack the plasticity of science and are now in imperfect contact with the contours of their new environment. The result, as events have shown, is friction and confusion. Though our civilization is based on science, the scientific method has little place in the making of our laws. Office does not seek the man in the laboratory, and candidates are not pictured as engaged in any activity that might suggest a superior intelligence. They are shown milking cows, pitching hay in new blue overalls, or helping with the family washing. Recently, in the senate of a New England state, there was presented the edifying spectacle of the presiding officer being shaved by a barber, called to the rostrum, while senators were reading the encyclopædia into the record. To expedite further the public business sundry members of the chamber were presently gassed with bromine. Does not this suggest that a few chemists might with advantage be distributed among our legislative bodies?

It is claimed that 50 per cent. of the members of state legislatures in America have never been through high school and that only one in seven has been through college. We see in the ranks of science knowledge without power and in politics power without

knowledge. An electorate, which regards itself as free, listens to the broadcast noise of manufactured demonstrations and is blind to the obvious mechanics of synthetic bedlam. The result is too often government by gullibility, propaganda, catchwords, and slogans, instead of government by law based on facts, principles, intelligence, and good will.

As President Stanley Hall once said, "Man has not yet demonstrated that he can remain permanently civilized." Many thoughtful people have been led to question the ultimate effect of science upon civilization. We all recognize the utility of matches, but we keep them away from children. Meanwhile, science puts dynamite and T.N.T., poison gas, aeroplanes, and motor cars at the disposal of criminals and the leaders of the mob. B. Russell, in "Icarus," sees in science the ultimate destroyer. Haldane, in "Daedalus," visualizes it as the stern and vigorous chastener and corrector which will ultimately save the race and usher in the new day of light and reason.

"Knowledge comes but Wisdom lingers," and democracy levels down as well as up. Even in Boston cigars have replaced books on a corner famous for a century of literary associations. The world is wrong because few men can think. It will not be made right until those who cannot think trust those who can. When its foundations are so obviously out of joint, humanity still clings tenaciously to fossilized precepts and opinions and is as resentful of suggested change as in the days of Galileo. Despite the pressure of new ideas, education must still, to be acceptable, follow old conventional lines.

Though we go from here to happy homes, let us not deceive ourselves. Human life is still a hard and fearsome thing. Mankind is required to maintain existence in a world in which, as Kipling has said, "any horror is credible." More than a hundred years ago De Quincey wrote, "We can die, but which of us, knowing as some of us do, what is human life, could, were he consciously called upon to do it, face, without shuddering, the hour of birth." But little more than yesterday Henry Adams closed his "Education" with the expression of the hope that perhaps some day, for the first time since man began his education among the carnivores, he would find a world that sensitive and timid natures could regard without a shudder.

Everywhere there is upheaval and unrest. "The machine,"

to quote Dr. Elton Mayo, "runs to an accompaniment of human reverie, human pessimism and sense of defeat."

We are everywhere overburdened by unnecessary illness, crushing taxation, extravagant and inefficient governments, huge expenditures for trivialities, and the appalling waste of effort, material, and resources. We are hampered by class suspicion and misunderstanding, racial antagonisms, the inhibitions of organized labor, and the lack of imagination in high places. Life in general is on a low cultural plane and bound by custom and tradition.

One hundred years of science have failed to satisfy the cravings of humanity. Chesterton finds science "a thing on the outskirts of human life—it has nothing to do with the centre of human life at all." We do not, of course, agree with him, but we must still meet the challenge of John Jay Chapman, who declares: "Science, which filled the air with so large a bray, is really a branch of domestic convenience, a department for the study of traction, cookery and wiring. The prophet-scientists have lived up to none of their prospectuses." The fault, however, as Wiggam points out, is not with science, nor with the scientists. It is with those who "have mainly used the immense spiritual enterprise of science to secure five-cent fares, high wages, and low freight rates," when it should have "ushered in a new humanism."

Thus we still encourage race deterioration, still carry the burden of the unfit, still cultivate national antipathies, still are breeding from poor stock and witnessing with equanimity the suppression of the best.

The history of aristocracies, feudalism, the church, the guilds, and the soviets has amply demonstrated that no one class possesses the qualities required for the government of all classes, and we cannot claim them for the Fifth Estate. We can, however, claim with full assurance that the Fifth Estate possesses many qualities, now practically ignored, which could be utilized in government to the incalculable advantage of us all. Its knowledge of material facts, of natural and economic laws, of the factors governing race development and human relations; its imagination, vision, and its open mind should be brought to bear effectively in the formulation of national policies and the solution of governmental problems. There is an alternative before us, which has recently been defined

with somewhat surprising frankness by Warren S. Stone, President of the Brotherhood of Locomotive Engineers, perhaps the most conservative of the labor unions. Mr. Stone says: "But until labor, in the inclusive sense in which I am using it, secures control of legislative and executive branches of the national and state governments, and through control of the executive branch secures control of the judiciary, labor is in continuous peril of seeing its gains wiped out and its progress retarded, by hostile legislation or unfriendly court decisions."

Our countrymen may well consider whether they prefer participation in government by the Fifth Estate to the benefit of all or control of government by labor unions in the interest of labor.

Since most of the troubles that beset mankind have their origin in human nature, it would seem worth the while of those who make our laws to study and apply the findings of the biologists and psychologists as to what human nature really is and what are the springs of its motivation.

Plato called democracy "the best form of bad government." It will be the best form of good government only as it develops the capacity to breed leaders and the faith to trust them. The quality of our children will determine the quality of our democracy. If our laws and mores and economic structure continue to discourage breeding from our best strains, if there is to be no adequate recompense for service of the higher types, the time is not far distant when democracy will no longer be safe for the world. If the Fifth Estate were everywhere to be wiped out, as it has been in Russia, the result would be vastly more calamitous than universal war.

Oswald Spengler, in a recent monumental work, forecasts the downfall of western civilization and would prove his thesis by the history of past cultures. But never in the past has man lived in so compact a world, never has he had such facilities for intercommunication with his fellows, never has he been endowed with such control of natural forces. He has never known himself so well and, above all, never before has he had it in his power to direct so definitely the course of his own development. Our civilization is certainly imperilled, but there will be no downfall if mankind can be taught to follow the light already before it. As lantern bearers, it is the clear duty of the Fifth Estate to show the way. In the

past the world has suffered grievously from lack of knowledge; to-day it suffers from its rejection or misapplication. Could the springs of human conduct and the affairs of peoples now be regulated only as wisely as we now know how there would be work and leisure and decent living for all. The criminal, the defective, the feeble-minded would be breeded out, and sane minds in sound bodies breeded in. The loss and suffering from preventable disease and accident would not be tolerated. Higher standards would govern the selection for the public service. Planning would replace *laissez-faire* development, and a rational conservation check the reckless waste of our resources. Production and distribution would attain to levels of efficiency altogether new, and the many injustices now existent in human relations would well-nigh disappear. With the reaction of a freed intelligence on politics, religion, and morals we might hope for a broader tolerance, a better mutual understanding. With the recognition of the spirituality of science and the divinity of research and discovery should come larger interests and a new breadth of vision to the average man, and to us all an acknowledgment of the steadfast, purposive striving shown in the development of the created world, together with a reverent appreciation of man's privilege to aid and further this development.

We might reasonably expect ugliness to be replaced by beauty in our cities and small towns and later even in our homes. Government by intelligence for the general good of all should supersede government by special interests, blocs, faddists, and fear of organized minorities and the uninformed crowd. With it all would come relief from the economic pressure, which bears so heavily upon the Fifth Estate that its children, which should be counted among the best assets of the community, are now a luxury.

The world needs most a new tolerance, a new understanding, an appreciation of the knowledge now at hand. For these it can look nowhere with such confidence as to the members of the Fifth Estate. Let us, therefore, recognize the obligation we are under. Ours is the duty and the privilege of bringing home to every man the wonders, the significance, and the underlying harmony of the world in which we live to the end that all undertakings may be better ordered, all lives enriched, all spirits fortified.

THE CARBON ATOM IN CRYSTALLINE STRUCTURE.*

BY

SIR WILLIAM HENRY BRAGG, K.B.E., M.A., D.Sc., F.R.S.

Royal Institution, London.

IT IS one of the great purposes of research to be able to explain the properties of materials in terms of the properties of their parts. The division into parts may be carried to various degrees of fineness; and the nature of the research, its methods and difficulties, and the expectations of success will depend on the division that is attempted. The properties of a piece of steel may to some extent be explained in terms of the small visible particles, crystalline or of which it is composed; the degree of division is determined by the power of the microscope, and results of great value are obtained within this range. But we may have, as a higher ideal, the hope of explaining the qualities of steel in terms of the properties of the atoms of iron and carbon and other constituent elements. The division is far finer than the other; but obviously far more complete and satisfying. The difficulties are great, but so also will be, we may be sure, the ultimate success. And generally it must be our aim to explain the properties of all materials in terms of the atoms, remarkably limited in kind, of which the world is made.

Some progress towards the ideal has been made in the case of the gas and the liquid states; the great effort is yet to be made in the case of the solid. The moving particles of a gas are free from each other's influence for the most part of their time; they can be treated as projectiles whose rare encounters are but slightly affected by the special and peculiar attractions which they exert on each other when very close together. The properties of gases are to a certain extent explainable in terms of a pure kinetic theory. Complications arise when from any cause the times during which the atoms or molecules are under each other's influence become relatively important, and the consequences depend on the individual peculiarities.

* Address delivered at the University of Pennsylvania, Thursday, September 18, 1924, on the occasion of the observance of the Centenary of The Franklin Institute.

So also in the case of liquids much can be explained on a purely hydrodynamical theory, especially when the ties between atoms and molecules can be dissolved and reformed continually without waste of energy in the form of heat, that is to say, when there are no viscosity effects. But when viscosity is to be taken account of, then the peculiar and individual actions of the atoms and molecules must be treated as effective. Still more, the phenomena of surface tension are direct manifestations of the forces that the atoms exert on their neighbors; and so in general are all the effects studied by the physical chemist. We are yet very far from being able to unravel them.

It is true that the associations and dissolutions of chemical action are due directly to the characteristic forces exerted by the atoms; and that chemical studies afford a most important means of examining them. Nevertheless, we want more than chemistry gives us if we are to reach our object. For instance, we are far from linking up the properties of quartz with the known chemistry of silicon and oxygen.

In general, we know very little of the relation between the properties of the solid material and of the atoms of which it is built. Of the relation between the molecule and its constituent atoms we do know something, thanks to chemical study. But the wide field of solid structure is in this respect almost unexplored. The reason for this is very simple. The properties of the solid depend upon the structural arrangement of its component atoms, which arrangement is at the first attempt, as we may say, crystalline. When groups of atoms or molecules associate and form a solid, they arrange themselves in a regular pattern; the unit of pattern containing only a few groups, usually two, three, four, six or other small numbers. The arrangement is so uniform and so exact as to imply that the group has a definite and characteristic shape, or, in other words, that the forces which one group, atom or molecule, exerts on its neighbors are such as to place them at proper distances from itself, and in proper orientations. The crystal is the direct expression of this tendency and is a far more prevalent form of matter than we had thought. The large crystals that we see with our naked eye, and even the small crystals which the microscope reveals to us, are only the favorable cases. Below all that we can see in any way is a universal tendency to crystallization which the methods of X-ray analysis are now

showing to us. The solid which appears to us to possess none of the properties of the crystal is usually a very viscous body, or a disordered mass of minute crystals. The crystal is the simple body, the so-called isotropic body is the complex. Consequently, it is the crystal that must first be mastered.

It is only within the last ten years that the direct study of crystal structure has been possible, and the way to this huge field of inquiry been thrown open. The X-rays, to put it simply, if somewhat crudely, are a form of light ten thousand times finer than visible light, and we are now able to see, indirectly it may be, the actual arrangement of the atoms. Here is our chance of making this supreme analysis. We have before us a widespread research, in the course of which we must examine crystal after crystal, learning its structure, that is to say, the pattern according to which its constituent atoms are arranged, and examining the way in which the properties of the individuals determine that pattern and in turn the pattern determines the elasticities, rigidities, conductivities, dielectric capacities and every other characteristic which the crystal possesses. After that, follows in its turn the crystal conglomerate, the isotropic solid.

An especial interest is attached to all that we can discover by the new methods of crystal analysis concerning the nature and properties of the carbon atom. It is not one of the more common elements. While oxygen is calculated to form half the materials of the earth's crust, silicon a quarter, and aluminum about a sixteenth, only about a thousandth is composed of carbon. But it is the carbon atom which more than any other impresses its character on the materials of which living bodies are composed. The very term, organic chemistry, which is given to the subject that is largely occupied with the study of the carbon compounds, is an expression of the importance of the carbon atom in the living organism. And not only is the carbon molecule of supreme importance in the constitution of our bodies and of their unconscious actions, but also in a great proportion of the external activities in which we are engaged. We must be especially eager to know what we may learn of the behavior of the carbon atom when it takes part in the constitution of the solid body, and in the first place of the crystal which is the simplest form of the solid. The only fact which makes us hesitate is the vastness of the number

of known crystalline compounds, giving us a bewildering choice of material on which to work.

We come to this study with certain expectations founded on the large labors of the organic chemist and their rich harvest of results. We expect the carbon atom to display, for example, a definite tetravalency, a power of attracting to itself under certain circumstances, at least four other atoms and no more, all bound in a similar way to the central carbon. As, for example, when the carbon atom draws to itself four atoms of hydrogen and the methane molecule is formed. Yet, under other circumstances, the carbon atom seems fairly content with a smaller number, as when in the benzene ring each carbon is attached to two other carbons and only one hydrogen, and the benzene molecule has very little general attraction for another benzene molecule. Again in the carbon dioxide molecule, the carbon atom is satisfied by the companionship of two other atoms only, and we should like to know which rearrangement has been made of its activities to permit the formation of this self-contained molecule, so independent that it forms a gas. And what further change has been made when the molecule of carbon monoxide is constituted? In other words, what is the explanation of the single, double, and triple bonds known to the chemist? We hope to be able to throw light on all these phenomena and the questions to which they give rise by the study of the carbon atom as it is built into the crystal where we can examine the exact nature of its environment and can relate thereto the properties of the crystal.

There are two crystalline forms composed of carbon alone, diamond and graphite, and we naturally turn to their examination first. The diamond structure was, in fact, one of the first to be examined by the new methods. Its structure is very simple and symmetrical, and answers at once our expectation that we should find under some circumstances a perfectly tetravalent behavior. Each carbon atom is surrounded by four others, which are grouped about it in perfect symmetry; the atom shows no attraction for more than four. A sphere can, of course, be surrounded by twelve other similar spheres, and if the attraction between any two atoms could be represented merely by a central force, we should expect the close packing which gives the full quota of neighbors. Since there are only four, the force exerted by one carbon atom on another cannot be represented by central forces

only. Further the hardness and rigidity of the diamond show us that the forces are mutually oriented; that they are primarily exerted along four lines drawn from the centre of a tetrahedron to its corners and that there is a strong resistance to any relative change in those directions. All these new points are in excellent agreement with known chemical facts. The carbon atom is to be represented by a tetrahedron rather than a sphere.

The disposition of the atoms in the diamond shows that they may be divided into two classes, any member of one class being the reflection of the other—with appropriate shift—in any one of three planes which are the cube faces of the crystal. To put this in another form, each diagonal of the cube crystal passes through the vertex and the middle of the base of each class of atom, but members of the two classes face opposite ways. The X-rays detect the difference between the two kinds, from which we conclude that the scatterers of the X-rays participate in the tetrahedral grouping. Yet the difference is only slight, so that if it is the electrons that scatter, they are either close to the centre which seems unlikely or are not closely pinned down to the corners, so that their diffracting effect would not be large; their action would be like that of broad faint rulings on a diffraction grating.

The other crystal of pure carbon is graphite. There is no clear evidence at present of the existence of any other form, although certain curious carbon materials have not been fully examined. The structure of graphite has recently been reexamined by Bernal, who was able to obtain single crystals of sufficiently perfect form. The results of his work show that graphite is a hexagonal crystal, as Hull had supposed from an examination by the Lowder method. It consists of layers of carbon atoms, each layer separated from the next by 3.40 ÅU, as has long been known. In each layer, the carbon atoms are arranged in a hexagonal network; the long uncertain point as to whether the atoms of one sheet are all exactly in one plane or are in two planes making a puckered network, seems now to be decided in favor of the former alternative, as Debye has suggested. In this case, each carbon atom is very strongly tied to three neighbors, all in one plane; the distance between the centres is 1.44 ÅU as against 1.54 ÅU in the diamond. The distance between a carbon atom in one layer and its nearest neighbor in the next is more than twice as great. The curious properties of graphite show that the

forces between atoms in the same sheet are exceedingly strong, while the forces between sheet and sheet are very small. It is clear that the carbon atom is now exerting its attraction on other atoms in quite a different manner from that which was followed in the diamond. If the electrons are in some way responsible for these forces their arrangement must have been altered. The atom is no layer tetrahedral.

The extraordinary success of Bohr's theory in regard to radiation is a constant inducement to attempt a correlation between his arrangement of electron orbits with the display of the atomic forces, although it does not seem possible at present to make more than vague and preliminary attempts in this direction. According to Bohr, there are four electron orbits which we may naturally associate with the tetravalency of the atom; but the four are not all alike, being different, two and two. Thus the tetravalency is not symmetrical. The radiating carbon atom which is the subject of the theory is not, however, attached to other atoms, but is free; and we may well suppose that the internal arrangements are modified by structural requirements. There may be a real difference, implying perhaps a different energy content between the free atom with its two pairs of outer orbits, the atom in graphite with three like orbits and one odd, and the atom in diamond with all four alike. The electron of the odd orbit in graphite would naturally travel far and be loosely attached to the nucleus; and in this, as has been suggested by Doctor Shearer and Mr. Bernal, we may find the explanation of the color, the opacity, and the conductivity of graphite.

We now come to cases in which other atoms beside the carbon enter into the structure; and first to calcite. The X-ray analysis shows that the carbon atom is now surrounded symmetrically by three atoms of oxygen, all alike.

The compact group of one carbon and three oxygens possesses a double negative charge; the calcium atom possessing the corresponding amount of positive electricity. It has been suggested by Kossel that the arrangement is governed by the tendency for the oxygen atoms to surround themselves in each case by the full complement of eight electrons, and that this is effected by stripping the calcium atom of two electrons and the carbon atom of four. On this theory, there is no parallelism with the graphite structure, though in both cases the carbon atom is surrounded by

three other coplanar atoms, similarly situated. The carbon atom has now lost all four of the electrons that were moving in outer orbits and is reduced, in external appearance, to helium. It is to be treated as exerting a central electrostatic force due to a positive charge of four units.

It is well known that a vast number of organic substances are based on a substructure consisting of a ring of six carbon atoms, or a chain of carbon atoms of any length. The simplest ring compound is the famous benzene molecule; benzene does not, however, lend itself very readily to analysis because benzene is liquid at ordinary temperature and when frozen does not form good crystals. But there are the two substances, naphthalene and anthracene, of which the former has been imagined by the organic chemist to consist of a double ring, represented by two hexagons in the same plane, having one side and two corners in common. The latter consists of three hexagons in a line, the naphthalene model extended by one more hexagon. Both these substances crystallize well, a very marked characteristic of each being the tendency to split into thin flakes. All these molecules are bounded by hydrogens. They must be considered as simple subjects for attack, because the ring is so common a feature of organic substances, and the single, double and triple rings form a series of comparable members. A curious point of obvious interest is the connection of each carbon atom with three, and only three, other atoms. Why has the fourth bond apparently disappeared?

The X-ray analysis shows that the unit of pattern of both naphthalene and anthracene contains two molecules arranged so that their long axes are parallel to one another and that they are the reflection of each other in the single plane of symmetry which the crystal possesses. In fact, the crystal can be considered as a set of parallel flakes, like the monomolecular layers of Langmuir, in each flake the molecules stand not quite at right angles to the flake, but leaning over like wheat in a field blown by the wind. The difference between the anthracene and the naphthalene structures lies only in the length of the molecule, which makes the flaky thickness of the former greater than that of the latter. Moreover, the actual increase in length which is found by the X-ray measurement is exactly what it should be if, in the first place, the one molecule consists of three benzene rings in a row and the other

of only two, according to chemical theory, and in the second place, the width of the ring is the same as that of the carbon ring which is found everywhere in the diamond structure. We might suppose, in fact, that the molecule of naphthalene as built into the crystal structure was simply carved out of the diamond without alteration and then fringed with hydrogen atoms. A similar set of atoms carved out of the graphite flake would do equally well so far as length is concerned, because the graphite layer is, according to Bernal, the diamond layer pressed flat without any sideways extension. But if the carbon atoms were in the supposed graphite condition, it might be expected that naphthalene would be a conductor and opaque like graphite, which is not the case. The direct analysis itself is not yet able to say whether the three attachments of each carbon atom to its neighbors in the same molecule are all on one plane. It is to be remembered that there are attachments between each molecule and its neighbors in the same layer, but these must be far weaker than the bonds binding together the atoms in the same molecules; they are stronger, however, than the end-to-end attachments which break on the cleaving of the crystal. These last must be feeble because the crystal cleaves so easily.

So far, therefore, the result of the analysis of these crystals is, in the first place, to give confidence in the existence of a mutual support between the established organic chemistry and the new methods of analysis; and to show us also how closely the behavior of the crystal may be connected with its general structure. As to details, the indication seems to be that the carbon atom has—at least nearly so—the same characteristics as the atom of the diamond; but that one of the four valencies is unused. We have even some grounds for saying that the unused valency, or the unoccupied corner of the carbon tetrahedron, is that which lies on the face of the molecule, not on its edge. The crystal of naphthalene tetrachloride, *i.e.*, naphthalene with four added chlorine atoms, shows the same structure as that of naphthalene, and has very similar dimensions except that it has broadened by an amount which would correspond closely with the result of adding the chlorines to the places indicated.

Measurements have been made of some of the dimensions of other crystals containing ring molecules, and the results fit in, as far as they go, with the idea that the ring is an actual structure

of definite dimension and form, slightly altered it may be by additions or substitutions of other atoms or groups of atoms, or strained because it has to be fitted into its place in the crystal, nevertheless recognizable in its different circumstances, as the chemist would expect. To settle these points more satisfactorily, to turn guesses into certainties is, of course, the work that is before us.

Very interesting results are obtained from the study of the long-chain molecules of the fatty acids, hydrocarbons, and similar substances. It appears that when these substances in the solid state are pressed onto a flat surface, they form flaky crystals like graphite or naphthalene, and it becomes easy to measure the thickness of the monomolecular layer. The experimental results are beautifully definite, and their interpretation can be given with great chance of being correct. It seems that the molecule is perpendicular to the layer, not slanting. If it were the latter, one would expect the amount of slant to be variable so that it would not be possible to connect the thickness of the layer with the length of the molecule only. Now, it is a remarkable fact that the thickness of the layer grows at a uniform rate as the chain is increased by the addition of carbon atoms and that for very nearly all, if not all, the different kinds of chain molecules that have been examined, this rate has one or other of two values. For every two carbon atoms that are added the increase in length is either 2.50 ÅU or 2.00 ÅU very nearly.

If we were to suppose that every carbon atom in the chain had four points on it, disposed like the corners of a regular tetrahedron, at any one of which an attachment could be made to a neighbor, in other words, if we supposed the atom to be as in diamond, then a chain of carbon atoms would take one or other of three forms, *A*, *B*, or *C*, the last of which is a screw. Putting the distance between the centres of two carbon atoms equal to the distance found in the diamond, *viz.*, 1.54 ÅU, it can readily be calculated that the length of chain *A* is 2.50 ÅU for every two atoms added, and of chain *B*, 2.05 ÅU. The agreement with experiment seems to be more than a coincidence, it is justifiable to assume that in these chains either the *A* or the *B* form is adopted, and the linkage of carbon atoms with their neighbors is on the diamond plan. In other words, if *PQR* are the centre of

three atoms, the angle *PQR* is equal to the angle between two of the lines joining the centre of a tetrahedron to the corners.

One of the most striking results is the fact that the length of the main body of the chain is independent of the nature of any additions to its ends or replacement of any of its side hydrogens by an oxygen atom as in the ketones. Even the removal of two or four hydrogens from the body of the chain leaving two carbons connected, respectively, by a double or a triple bond, makes no appreciable difference in the length. It would not have been a matter of surprise if such treatment had caused a bend in the chain, on account of the relative shifting of points of attachment on the carbon atom that might be due to other points of attachment being unused. The removal of a pair of hydrogens from the chain makes no more difference to the chain than the stripping of an opposing pair of leaves from the stem of a plant.

A very curious point in the behavior of the long chains is their apparent doubling in length when one end is formed of a so-called active group, as for example, in the case of the fatty acids. When this is the case two chains join end to end, the active groups being together, and again the result is just the same as if two sticks were joined into one, the process of joining being concerned with nothing beyond the two active ends. This form of structure is illustrated in a very interesting way in the X-ray spectra. If a section is made of a substance consisting of layers of these doubled molecules, so as to show their stratification, then there will be a uniform distribution of diffracting centres—electrons—throughout the mass, with alternating thin layers of excess through the active ends, and thin layers of deficiency through the inactive ends, the methyl groups. This has the curious effect of intensifying the odd orders of the diffraction spectra. To understand this, it is only necessary to remember that if we were to make a series of fine rulings on a diffraction grating which exactly interleaved the stronger original rulings, the result would be the strengthening of the even orders; such an effect is found in the case of rock salt and was an important help in the determination of its structure. If now the additional rulings, instead of being like the original but weaker, could be of reversed effect, a deficiency instead of an excess, it would be the odd, not the even order that would be strengthened. The same effect is found in the case of the ketones. When the substituted oxygen is in the middle of the chain, each

layer of hydrocarbon molecules shows a generally uniform distribution of electrons with an excess at the middle where the oxygen atom is placed and a deficiency at either end. When the substituted oxygen is not in the middle, the reinforcement of the odd orders disappears.

These many observations of the behavior of the long-chain molecules seem to point to the conclusion that the carbon atom in the chain is, like the carbon atom in the diamond atom, possessed of four points of attachment regularly disposed about it, and that the non-use of one does not alter the disposition of the others.

It is scarcely necessary to consider in any detail the points given by structures like tartaric or succinic acid as to the behavior of carbon atom. It is enough to say that, while the full interpretation is yet to come, the general indications are in accordance with what has already been said.

I have thought it might be of interest to consider these first tentative conclusions as to the behavior of the carbon atom in the solid body, in spite of the fact that so little, relatively, has been accomplished in the solution of the structure of organic crystals. What is to be done must have great consequences, and what little has been done already seems to possess many points of novelty and of interest, and these two facts taken together may be sufficient justification for my attempt at a review of the present position of the inquiry.

Bohr's Atomic Theory and Electrical Conductivity. O. FEUSSNER. (*Zeit. f. Physik*, Vol. 25, No. 3.)—The author presents a plot of the elements arranged in a line according to their atomic numbers with their electrical conductivities as coördinates. Along with this is a table by Bohr which shows how the electrons are distributed in the various orbits about the nucleus. The author gives a supplementary table of his own and states the rule that the greatest conductivity is found in those elements which have one electron only in the outermost ring.

It is quite interesting to glance through Bohr's table and pick out the elements that have this characteristic—H, Li, Na, K, Cu, Rb, Ag, Cs, Au, with the element of atomic number 87—and then to turn the eye to the curve of conductivities where Na, K, Cu, Rb, Ag, Cs and Au occupy maximum points. The agreement is by no means as satisfactory if the results of the inspection of the author's table be compared with the curve. When element number 87 is discovered it will be expected to have a high conductivity. G. F. S.

The Division of the Mercury Atom. A. MIETHE, Photochemical Laboratory, Technische Hochschule, Charlottenburg. (*Naturwissenschaften*, July 18, 1924.)—In itself the title of this preliminary communication is quite attractive, but when it is said that gold results from the breaking up of the mercury atom the article assumes an unusual interest. For years Professor Miethe has been studying the changes of color produced in transparent minerals and in glass by the action of ultra-violet light. As the source of this kind of radiation he has used the mercury-arc lamp. In June, 1923, he made the acquaintance of the Jaenicke lamp, which he later found to lessen rather quickly its ultra-violet output by reason of a dark deposit on the interior of the tube. After a discussion of this defect of the lamp with its inventor the latter stated that from the distillation of mercury from old lamps there remained a residue whose chemical character he could not determine. Professor Miethe himself analyzed such a residue and detected gold. He then started out with electrodes and other lamp parts free of gold and with mercury that gave no trace of the metal when tested by more than one analyst and by more than one method. The current was then passed through the mercury vapor for a period of from 20 to 200 hours, the voltage being near to 170 with an expenditure of 400 to 2000 watts. After this the mercury in the lamp was distilled and in the residue gold was present. That it was really gold was confirmed by several tests. The mass seems to have been of the order of .001 to .1 milligram.

The author holds that a certain minimum potential difference is necessary for the production of gold and in this way accounts for the absence of gold in the mercury of some commercial lamps that he examined.

The atomic weight of mercury is about 201. If from this be subtracted 4, which may be regarded as the weight of an atom of helium or of four atoms of hydrogen, the remainder, 197, is the atomic weight of gold, the element whose atomic number is one less than that of mercury. Unfortunately for our curiosity Professor Miethe apparently made no attempt to detect the formation of helium or hydrogen by the same process that produced the gold. He promises further investigations. Will these show that he is in error or will they demonstrate that the dream of the alchemists is at last realized?

G. F. S.

On the Measurement of Very Small Changes of Capacity. Ross GUNN. (*Phil. Mag.*, July, 1924.)—Two methods are given, both of which include the use of a vacuum tube. In the first a change of one microampere of current is caused by a change of 10^{-15} farad capacity. The second is so sensitive "that the approach of a person within several feet produced a current change of a few microamperes. It was easily possible to obtain a sensitivity of one microampere for a capacity change of the order of 10^{-18} farad."

G. F. S.

PRELIMINARY MEASUREMENT OF THE VELOCITY OF LIGHT.*

BY

A. A. MICHELSON, Ph.D., Sc.D., LL.D.

University of Chicago.

THE velocity of light is one of the most fundamental of the constants of Nature, and this fact alone would justify the attempt to measure its value with the highest possible precision. But in addition to its scientific importance, it may prove to have a practical value if the result of such a measurement can be obtained with sufficient accuracy.

The mean of the various measurements thus far attempted is 186,330 miles per second, with an uncertainty of twenty or thirty miles. If this uncertainty could be reduced to one mile per second, the timing of light could be utilized to obtain distances between stations from 50 to 100 miles apart far more expeditiously and with an order of accuracy at least as great as that obtainable by the usual method of triangulation. Indeed, there are possibilities of utilizing the velocity of light in cases where triangulation would be difficult or impossible.

An invitation tendered by Dr. G. E. Hale, then Director of the Mt. Wilson Observatory, and supported by Dr. J. C. Merriam, Director of the Carnegie Institution, made it possible to install the necessary apparatus on Mt. Wilson, with Mt. San Antonio, twenty-two miles away, as the distant station, during the summer of 1923; but smoke and haze from burning oil and from forest fires made it impossible even to test the feasibility of the method at so great a distance.

This feat was accomplished during the past summer with very promising results. The set-up of apparatus involved several important changes in the arrangement employed in previous investigations, the most important of which consisted in the substitution of an octagonal revolving mirror for the usual plane-parallel, together with the introduction of a system of reflectors which eliminated all direct and diffuse extraneous light. Finally, a simple method for returning the light from the distant station to

* Address delivered at the University of Pennsylvania, Friday, September 19, 1924, on the occasion of the observance of the Centenary of The Franklin Institute.

the source was substituted for the plane mirror used for this purpose in previous work, and this equipment functioned so well that no readjustment was required during the entire two months of the work.

The advantage of the octagonal revolving mirror, in addition to the higher speed obtainable, lies in the possibility of receiving the return light on a succeeding face, thus eliminating the measurement of the angular deflection of the returned beam; or rather, transferring this measurement to the construction of the octagon, the angles of which were tested and found to be equal, with an uncertainty of only one part in a million.

The determination of the velocity of light is thus reduced to the measurement of the distance between the stations, and of the speed of rotation of the mirror. The former operation was carried out by the U. S. Coast and Geodetic Survey, and the result obtained was 35,426.3 metres (about twenty-two miles), with an uncertainty of the order of only two parts in a million. The errors in the measurement of the speed of the revolving mirror were much greater, as no very effective means were employed to insure its constancy. (This defect will be eliminated in the continuation of the work next summer.)

Notwithstanding the inconstancy of the speed of the mirror, the choice of the most favorable moment, when the speed was that corresponding to the frequency of a control tuning fork, made the resulting uncertainty of the measurements of the order of one ten-thousandth part, which is about that of the mean of all the previous measurements. It is hoped that next year's work will furnish results four or five times more accurate.

The result of eight independent observations in the present preliminary work is, for the velocity of light in vacuo, 299,820 kilometres per second.

Following is a table of results of the more important investigations to date, with an estimate of the weight which should be assigned to each:

Investigator.	Method.	Distance.	Weight.	Velocity.
Cornu	Toothed wheel	23 kilom.	1	299,950
Perrotin	Toothed wheel	12 kilom.	1	299,900
Michelson	Revolving mirror	0.6 kilom.	2	299,895
Newcomb	Revolving mirror	6.5 kilom.	3	299,860
Michelson	Revolving mirror	35.4 kilom.	3	299,820

STUDIES IN PHOTOGRAPHIC SENSITIVITY.*

V. ACTION OF ARSENITE AND OXIDIZING AGENTS ON THE SENSITIVITY.

BY

S. E. SHEPPARD, E. P. WIGHTMAN, and A. P. H. TRIVELLI.

Research Laboratory, Eastman Kodak Company.

THE earlier work of Bancroft,[1] Perley[2] and others on the effect of sodium arsenite on the photographic plate, has recently been extended by Walter Clark.[3] He has stated that commercial sodium arsenite (Kahlbaum) does not react with silver bromide. He titrated a 1 per cent. solution of sodium arsenite, which has been shaken with washed and precipitated silver bromide, with a standard iodine solution, by Washburn's method, and obtained the same titration value as with the arsenite to which no bromide had been added.

From this result and that of some previous work[4] he drew some important conclusions regarding the mechanism of the action of arsenite on the photographic plate. He thinks that the grains of the plate contain sensitive centres present in the grains prior to exposure, which consist of Ag_2O or some other easily reducible silver compound (other than AgBr), and that the arsenite reduces this compound to silver.

The idea of pre-exposure sensitive nuclei was not a new one,[5] but the evidence brought forward by Clark was more specific than most previous results. We agree with him that there probably are such pre-exposure centres, but we fail to see how his iodometric determination clenches his conclusions.

---

* Communicated by Dr. S. E. Sheppard, Acting Director, and published as communication No. 211 from the Research Laboratory of the Eastman Kodak Company.

An abridgement of this paper was presented at the Washington meeting of the American Chemical Society, April, 1924.

[1] W. D. Bancroft, *J. Phys. Chem.*, **14**, 292 (1910).

[2] G. A. Perley, *Ibid.*, **14**, 685 (1910).

[3] W. Clark, *Phot. Jour.*, **63**, 230 (1923).

[4] W. Clark, *Brit. J. Phot.*, **69**, 462 (1922).

[5] For discussion and review of this see paper by Sheppard, Trivelli and Wightman, JOUR. FRANK. INST., Nov. and Dec., 1923 (No. 111 of this series).

In the first place, it is conceivable that a quantity of silver arsenic complex salt of the order of magnitude of the latent image might be sufficient to cause developability. If such an amount were formed it could not be detected by the method which Clark used.

Secondly, it appeared to us that if silver bromide is dissolved by monosodium arsenite solution and reacts to form a silver complex arsenite, then one should not be able to detect this by titration with iodine, since the total content of arsenite should remain the same. On the other hand, if silver arsenite be added to the solution of monosodium arsenite, the total arsenite content would be changed and a titration with iodine should show this, unless the quantity added be within the limits of experimental error.

A 10 per cent. solution of monosodium arsenite [6] was made in freshly boiled distilled water from very pure As_2O_3 and NaOH. This solution was divided into five parts, which were treated as described. For the titration of each, 10 c.c. were diluted to 100 c.c., and 20 c.c. of this were titrated by the method of Washburn [7] with approximately N/10 iodine (0.1024 N to be exact). In each case the 20 c.c. of arsenite mentioned refer to the diluted solution.

(1) 20 c.c. arsenite titrated immediately $\simeq$ 26.15 c.c. Standard I solution.
(2) 20 c.c. arsenite after 20 hours $\simeq$ 26.00 c.c. Standard I solution in vessel with air excluded.[8]
(3) 20 c.c. arsenite saturated as nearly as possible with AgBr, and the clear supernatant liquid titrated $\simeq$ 25.97 c.c. Standard I solution.
(4) 20 c.c. arsenite saturated with Ag arsenite and the clear supernatant liquid titrated $\simeq$ 25.99 c.c. Standard I solution.
(5) 20 c.c. arsenite + 0.02 c.c. N/10 $AgNO_3$ (just below point of precipitation and $\simeq$ 0.2 mg. Ag) $\simeq$ 25.94 c.c. Standard I solution.

In case (4) we had expected the titration value to be greater than in (2) or (3), because we had actually added arsenite, but

[6] Clark used a 1 per cent. solution of arsenite for his iodometric determinations, but a 10 per cent. solution for the treatment of his plates. Since the solubility of silver arsenite is not the same in the dilute as in the concentrated solution, his procedure seems to us unwarranted.

[7] E. W. Washburn, *J. Am. Chem. Soc.*, **30**, 21 (1908).

[8] When no precautions are taken as to boiling the water used for the solutions, and as to exclusion of air from the vessel in which the arsenite is allowed to stand, the difference in the titration value, on standing, from the fresh material is considerably greater than this titration value, showing that there is a fairly rapid oxidation of arsenite to arsenate.

after determining the amount of silver present in such a solution by other means (namely, 0.1 to 0.2 mg. in the 20 c.c.) it was found that the maximum difference in the titration value should be less than 0.08 c.c., exclusive of experimental and other sources of error. The facts that the silver arsenite added to the sodium arsenite solution was not perfectly dry; that there is some oxidation of the sodium arsenite while saturating with silver salt and during other manipulations; and that there is a partial decomposition of the silver arsenite itself, could easily account for this small difference.

Again in (5), the titration value is slightly less than in (2) or (3), although we know that a silver salt equivalent to about 0.2 mg. of silver is present in the 20 c.c. of diluted solution.

When As_2O_3 and NaOH are mixed so that the proportion of Na to As is 2 to 1, and this solution is saturated with silver arsenite, a considerable amount of silver salt is dissolved. In making an iodometric titration, a portion of the clear filtrate from this is diluted as above, but a large portion of the silver salt precipitates. If the whole mixture of solid and liquid is used for the titration, then the results for 20 c.c. of the diluted mixture are—

20 c.c. disodium arsenite $\simeq$ 22.83 c.c. Standard I solution.
20 c.c. disodium arsenite + Ag arsenite $\simeq$ 24.10 c.c. Standard I solution.

On the other hand, when 6.5 c.c. of N/10 $AgNO_3$ (equivalent to 2 mg. Ag in 20 c.c. of the final diluted solution) are added to 100 c.c. of the disodium arsenite solution and a portion of this is diluted and titrated as above, in comparison with a portion from another 100 c.c. sample to which 6.5 c.c. of distilled water have been added, we get:

20 c.c. disodium arsenite + $AgNO_3$ $\simeq$ 21.8 c.c. Standard I solution.
20 c.c. disodium arsenite + H_2O $\simeq$ 21.8 c.c. Standard I solution.

It is seen, therefore, that when silver arsenite is added to the sodium arsenite solution in sufficient excess of original concentration of sodium arsenite, it is easily determinable by the iodometric method, whereas when the silver arsenite is formed in the solution from the sodium arsenite originally present, it cannot be separately detected iodometrically in any concentration. We feel, therefore, that this, with what follows, is sufficient proof that Clark's contention is incorrect.

It may be of interest to know the amount of silver nitrate which can be added to sodium arsenite solutions containing one, two and three theoretical equivalents of sodium. A 0.1 per cent. (about 0.06 N) $AgNO_3$ solution was titrated slowly with stirring into 10 c.c. of 5 per cent. arsenite solution until a very faint permanent cloudiness appeared. The quantities added are given in Table I.

If more than the above quantities of $AgNO_3$ is added to the monosodium solution, a light yellow precipitate of silver arsenite or complex salt is formed. The precipitates from the arsenites of higher sodium content have a deeper yellow color, but in no case is a brown or black precipitate formed at the start. The precipitates from the higher sodium salts, however, quickly become

TABLE I.

10 c.c. of 5 per cent. Arsenite Solution Containing	No. of c.c. of 0.1 per cent. $AgNO_3$ Added until Faint Cloudiness.	Gm. $AgNO_3$ Added per 100 c.c. of Mixture.
1 Na	1.13	0.009
2 Na	13.06	0.057
3 Na*	16.13	0.062

*The 1, 2 and 3 represent not atomic quantities, but proportions.

blackened by deposition of silver formed by decomposition of the silver salt in the solution.[9]

No quantitative determination was made as to the composition of these precipitates, but a qualitative test showed them to contain an abundance of arsenic as well as silver.

The approximate amount of silver bromide which is absorbed by the three arsenite solutions has been determined indirectly as follows: Silver bromide was precipitated from water solutions of $AgNO_3$ and KBr in molecular proportions, in dull yellow light (Wratten Safelight No. 0), and was filtered, washed and dried in air (in the dark) at room temperature. A weighed quantity of the dry salt was then added to a measured quantity of the arsenite solution, and this was mixed in a "tumbling machine" during twenty-four hours or more. At the end of this time the mixture was filtered and the residue was washed with distilled water, dried

[9] This decomposition takes place even when no solid salt is suspended in the solution.

and weighed. The residue was not pure AgBr, but contained also metallic silver, which we assume was formed by the decomposition of the silver arsenite in the solution, since the portion of this substance which remained in solution, after filtering off the AgBr and metallic Ag, continued to decompose and deposit silver. This spontaneous decomposition is an important fact, which we shall mention later.

The silver was dissolved out of the residue by means of HNO_3 (50 c.c. of concentrated acid to 50 c.c. of water) and the AgBr was again washed, dried and weighed. The residue from the monosodium solution was found to contain very little if any silver,

TABLE II.

	Solution Containing 1 Na: 1 As.	Solution Containing 2 Na: 1 As.	Solution Containing 3 Na: 1 As.
Weight of AgBr added per 100 c.c. of 10 per cent. arsenite solution...	2.0048 g.	4.0040 g.	4.0010 g.
Weight of residue (AgBr + Ag) after twenty-four hours	1.9908	2.5374	2.1094
Weight of AgBr residue free of Ag.	1.9875	1.3154	0.4454
Weight of AgBr absorbed	0.0173*	2.6886	3.5456

*The solubility of AgBr in water is about 0.000026 g. per 100 c.c. at 25°.

while the percentage of silver in the other two was quite high. By subtraction of the weight of AgBr residues (free of Ag) from the original weights of AgBr added, the amounts which dissolved and reacted with the arsenites were determined. The data are given in Table II.

The most direct proof of the fact that silver bromide is dissolved by the monosodium arsenite solution is that silver can be found in the solution. To 20 c.c. of the clear arsenite solution, approximately saturated with AgBr as above described, was added about 1 c.c. of hypophosphorous acid as a reducing agent. There resulted a milky cloud of silver in suspension. A blank with sodium arsenite solution containing no bromide showed no trace of cloudiness; and tests with the arsenite to which known quantities of 0.1 N $AgNO_3$ had been added showed that the amount of silver salt present, calculated as silver, was of the order of 4 mg.

Ag per 100 c.c. of the solution, *i.e.*, about 6 mg. of $AgNO_3$ were dissolved. Tests with the monosodium arsenite saturated with silver arsenite indicated a silver content of approximately 6 mg. per 100 c.c., which corresponds to about 12 mg. of $AgAsO_2$.

Since silver nitrate reacts with the monosodium arsenite to give a silver arsenite compound of some kind, and since a quite comparable amount of silver bromide dissolves in the arsenite, it is presumable that this also reacts to form a complex silver arsenite, but, as stated, no effort was made to determine its composition.

Following up a criticism of his work by Lüppo-Cramer,[10] Clark [11] carried out experiments which he said show that the more alkaline arsenite salts are complexes with silver bromide, but that no complexes are formed with the monosodium salt $NaAsO_2$.

Some of his other conclusions we shall mention later.

We do not agree with Lüppo-Cramer that the experiments with arsenite are of *no* value in helping to solve the problem of the latent image and sensitivity. Yet, on the other hand, we believe him justified in some of his criticisms, and our experiments uphold his view that the fogging by arsenite treatment followed by development " is probably due to a reaction of the silver halide with the arsenite with the formation, perhaps only in traces, of a complex salt which is readily reducible "—so much so, indeed, that it deposits silver spontaneously, without the addition of developer, probably on the sensitive nuclei, making them developable. But more of this later.

Clark reviews the literature of the interaction of arsenite solution with silver compounds and also gives equations which he says " represent some of the possible reactions in solution between caustic soda and arsenious oxide."

Curiously enough, he fails to mention the most authoritative work so far done on the subject of arsenites, namely, that of Schreinemakers and de Baat.[12] They have shown that among others, the following salts exist:

(1) $NaAsO_2$

[10] Lüppo-Cramer, *Phot. Ind.*, **40**, 456 (1923).

[11] W. Clark, *Brit. J. Phot.*, **70**, 456 (1923).

[12] F. A. H. Schreinemakers and Miss W. C. de Baat, *Rec. trav. chim.*, 39, 423 (1920); *Verse. Kon. Akad. v. Wetensch. Amsterdam*, Feb. and May, 1915; *Chem. Weekbl.*, 141, 203, 244, 262, 283 (1917).

which is merely the theoretical monosodium salt NaH_2AsO_3, less one molecule of water. In solution it is quite probable that the water is associated with the ions or molecules of the salt.

(2) $Na_4As_2O_5.9H_2O$

which they say may be written as $Na_4H_2As_2O_6.8H_2O$ or $2Na_2HAsO_3.4H_2O$.

There does not appear to be any salt of the theoretical acid H_3AsO_3. The nearest to it would be the salt

(3) $Na_6As_4O_9.12H_2O$.

The authors do not list this salt, but they do the potassium salt corresponding to it. It is presumable, however, that the sodium salt also exists. According to the authors, an equivalent way of writing this salt is $(Na_3AsO_3)_2As_2O_3.12H_2O$.

Finally, they list one other complex salt of still higher sodium content with respect to arsenic

(4) $Na_{16}As_4O_{11}.26H_2O$

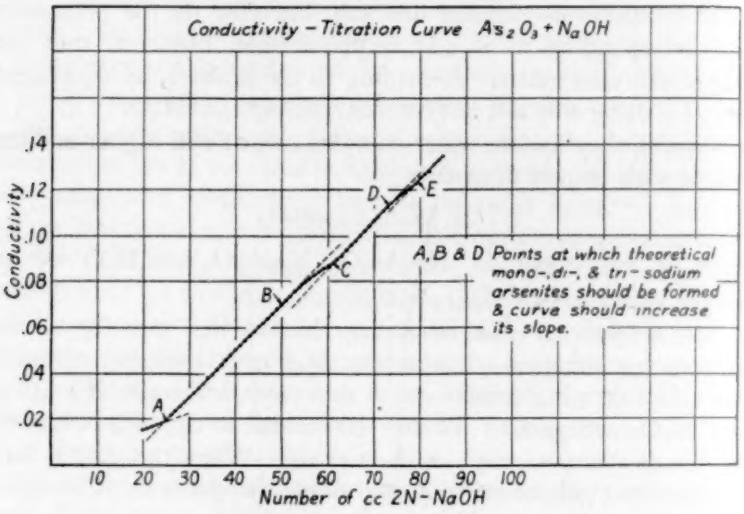
which is equivalent to $Na_4As_2O_5(Na_3AsO_3)_2.26H_2O$ or to $(Na_2HAsO_3)_2(Na_3AsO_3)_2.25H_2O$.

Mr. Crouch of this laboratory has carried out for us an electrometric titration of arsenious acid with sodium hydroxide under the following conditions: A thin paste was made of 4.948 g. pure As_2O_3 with water. To this were added 20 c.c. of 2 N NaOH and the mixture warmed, with stirring. When the As_2O_3 had entirely dissolved the solution was diluted to 500 c.c. This solution corresponded to N/10 solution of the theoretical acid $[H^+, (H_2AsO_3)^-]$, or $[H^+, (AsO_2)^-]$. Twenty-five c.c. of 2N NaOH would be sufficient to give the monosodium salt of this acid, but the As_2O_3 will go into solution completely, before this amount is added, contrary to Clark's statement. The conductivity of this solution was then read at 25°, and again after each successive addition of 1 c.c. of the NaOH solution. The solution was vigorously stirred mechanically during the whole operation.

At the point on the curve corresponding to the formation of the monosodium salt, formula (1) above (*A* in Fig. 1), there is a distinct break, with an increase in the slope of the curve, as there should be theoretically. Addition of NaOH then gives a straight line to the point where the next break should occur if the theoreti-

cal substance Na_2HAsO_3 were to be formed. But there is no break followed by an increase in slope of the curve from this point. Instead, the curve bends very gradually downward for a period and then continues in another straight line parallel to the preceding one. This may be taken as indicating the formation of a more complex salt than the simple divalent [2 Na^+, $(HAsO_3)^{--}$], with a consequent removal of ions from the solution. As soon as no more of the complex salt is formed, the curve then bends upward and follows a straight line with further addition of NaOH, until

FIG. 1.



the next point at 3 equivalents of Na is reached, where the same phenomenon occurs.

The formation of such a compound as that in formulas (2) and (3) above could account for this result, although we shall not here speculate as to just what would be the mechanism of their formation and ionization. Whether or not the three salts above exist as such in solution, we do not know. Also little is known as to what are the silver salts corresponding to them. We agree with Clark that a systematic investigation into this problem is very desirable. Although a fruitful field for research, the problem of the sensitivity of the photographic plate does not depend upon its solution, so we have not investigated it further.

Clark's experiments with commercial and standard arsenite solutions [13] are somewhat confusing, and the object of some of them is not quite clear to us. Also, others do not seem to be in entire agreement with experiments of a similar nature which we have carried out, following the work of Sheppard [14] with complex silver sulphite solutions.

Table III gives a series of such experiments which we tried with the idea of seeing how easily silver would be precipitated from arsenite solutions containing a silver salt (arsenite or some complex).

It will be observed from the data that while addition of colloid silver tends slightly to increase the rate of decomposition of the silver arsenite in solution and addition of sodium sulphite tends somewhat to diminish it, that even with no addition of any kind, there is a slow spontaneous decomposition of the silver compound, increasing in speed with the increasing alkalinity. With developer the reduction is almost instantaneous.

Colloid silver itself tends to precipitate in the presence of the sodium arsenite. This fact is contrary to Clark's statement that the tendency of the arsenite solution is " to peptize the silver nuclei formed in the earlier stages of treatment and so bring about a decrease in the reduction centres."

Seed 23 plates have been treated in steps for varying lengths of time with 5, 10 and 25 per cent. arsenite solutions containing one, two and three sodiums for one arsenic. Some series were developed in amidol developer as used by Clark and others in ordinary 1 : 1 pyro developer. Photomicrographs were made in every case.

It is not worth while going into detail concerning all of these experiments. The principal facts brought out were:

(1) For this emulsion, even with drastic development of plates treated with any of the arsenites for periods of time ranging from ten seconds to eighty minutes, no great developability was produced.

(2) The solubility of the AgBr grains in the monosodium salt, even in the 10 per cent. concentration, and for the longest periods of treatment, was *to the eye* very slight. There was considerable solubility of the grains in the 10 per cent. arsenites of stronger

[13] *Loc. cit.*, pp. 717–718.

[14] *Phot. Jour.*, **43**, 136 (1919).

TABLE III.

	Na:As	Na_2:As	Na_3:As
Added 1 c.c. colloid Ag (0.02 g. per 100 c.c.) diluted to 10 c.c. with H_2O to	2.5 c.c. 10 per cent. arsenite	2.5 c.c. 10 per cent. arsenite	2.5 c.c. 10 per cent. arsenite
Color change:	Faint orange to violet	Faint orange to red	Faint orange to red
After twenty-five hours standing:	Ag precipitated	Ag precipitated	Ag precipitated
Color of precipitate:	Blue-black	Brownish	Bright red
Added 5 c.c. colloid Ag (0.02 g./100 c.c.) to	5 c.c. 10 per cent. arsenite	.5 c.c. 10 per cent. arsenite	5 c.c. 10 per cent. arsenite
Color change:	Same as before	As above	As above
After six hours:	Fine grains Ag coagulated, but in suspension		
After twenty-four hours:	Ag completed, precipitated	No perceptible precipitate	No perceptible precipitate
After ninety-six hours:		No perceptible precipitate	No perceptible precipitate
Saturate arsenite solutions with AgBr in presence of excess AgBr			
On standing one hour:	No apparent change (see Table II above)	AgBr becomes gray-black with precipitated Ag	AgBr becomes black with precipitated Ag and mirror forms on walls of vessel
A duplicate filtered			
On standing twenty-four hours:	No apparent change *	Slight gray-black precipitate of Ag	Slight gray-black precipitate of Ag and Ag mirror

TABLE III (*Continued*).

	Na_1:As	Na_2:As	Na_3:As
On heating to 80°:	No apparent change *	Black precipitate of Ag immediately	Black precipitate Ag immediately and silver mirror
A duplicate filtered			
To this add:			
(1) 0.5 c.c. colloid Ag solution—			
On standing one hour:	No visible precipitate	Slight gray precipitate (more than colloid Ag)	Heavy black precipitate and mirror on walls of vessel
On standing twenty-four hours:	No visible precipitate *	Darker gray	Heavy black precipitate and mirror on walls of vessel
On heating to 80°:	No visible precipitate *	Immediate precipitation of Ag	Immediate precipitation of Ag, mostly as mirror
(2a) 0.5 c.c. colloid Ag solution + 5 c.c. 10 per cent. Na_2SO_3			
On standing one hour:	No visible precipitate	Slight black precipitate	Heavy black precipitate
On standing twenty-four hours:	No visible precipitate *	Precipitate increased	Heavy black precipitate
On heating to 80°:	No visible precipitate *	Yellowish-white precipitate	Black precipitate and mirror immediately
(2b) 5 c.c. 10 per cent. Na_2SO_3 and no colloid Ag			
On standing one hour:	No visible precipitate	No visible precipitate	Slight black precipitate
On standing twenty-four hours:	No visible precipitate	Slight precipitate	Medium precipitate

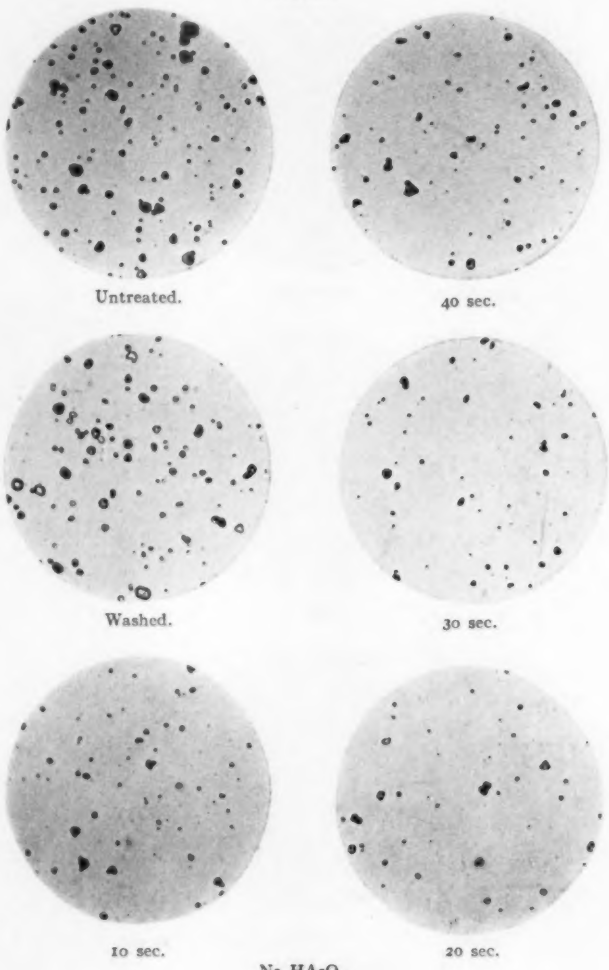
TABLE III (*Continued*).

	$Na:As$	$Na_2:As$	$Na_3:As$
(*3a*) 0.5 c.c. colloid Ag solution + 2 c.c. 1:1 pyro developer	Light gray precipitate immediately	Heavy brown precipitate	Heavy dark brown precipitate
On standing twenty-four hours:	Black precipitate settles	Turns grayish-brown	Heavy dark brown precipitate
(*3b*) 2 c.c. 1:1 pyro developer	Very slight light gray precipitate	Heavy brown precipitate. Less than in (*3a*)	Heavy dark brown precipitate. Less than in (*3a*)
On standing twenty-four hours:	Very slight black precipitate settles	Turns grayish-brown	Heavy dark brown precipitate. Less than in (*3a*)

*A larger quantity of the mono-arsenite solution saturated with AgBr, when evaporated only comparatively little in a vacuum oven at 40°, precipitates silver, that is, silver arsenite does not crystallize out as would be expected, but decomposes.

alkalinity, as shown in Figs. 2 and 3 (the formulas are only imaginary and given for convenience to show the relative quantities of sodium to arsenic). In 5 per cent. solution, however, the solu-

FIG. 2.

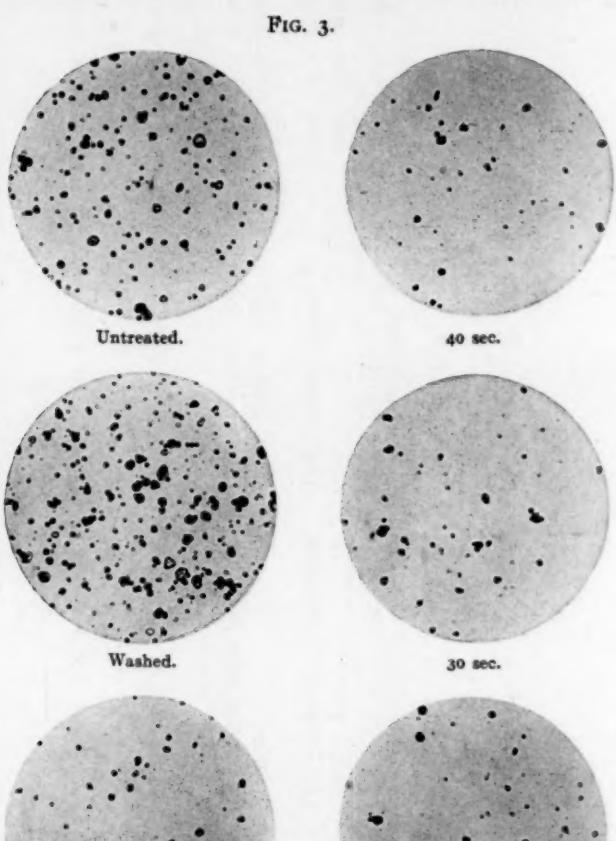


Na_2HAsO_3
10 per cent.

bility is much less, and in the 2.5 per cent. solutions is not perceptible to the eye. The lack of developability is likewise shown in these figures.

(3) When the arsenites are saturated with AgBr before treatment of the plate, for short times of treatment, up to forty seconds, very little if any solubility of the grains is apparent, even with the

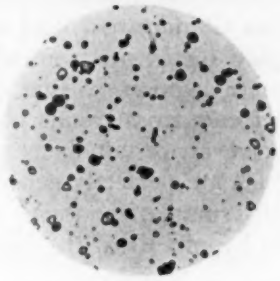
FIG. 3.



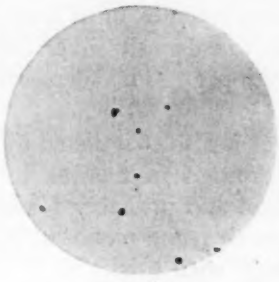
most alkaline arsenite in 10 per cent. concentration, and only slight developability is produced on (Seed 23) plates. The same is true for longer times of treatment with the monosodium salt saturated

with AgBr. With the more alkaline solutions, however, after the Ag arsenite in the solution has had a chance to decompose to any great extent and deposit silver, the silver halide grains then go

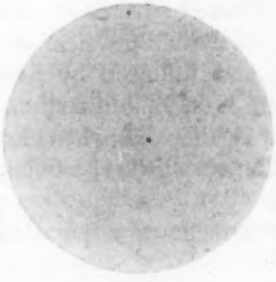
FIG. 4.



Untreated.



10 min.



40 min.

$Na_2HAsO_4 + AgBr$
10 per cent.

into solution and react to form silver arsenite, to replace that which has been decomposed. (Fig. 4.)

It should be noted here that plates treated with Merck's C. P.

"Sodium Arsenite" showed the same solubility of the silver halide grains as with the disodium salt solution which we prepared from As_2O_3 and NaOH. No formula was given on the bottle. Clark's results with commercial arsenite, it would appear, did not show this. Very likely, his arsenite was the monosodium salt.

It is apparent from Clark's work and our own that various brands of photographic plates are somewhat specific in their behavior toward arsenite solution, so that one should be careful not to generalize too freely from results obtained from any one brand. There are, however, certain facts and conclusions to be drawn from them which are perfectly general to all silver halide emulsions. We shall summarize these here, and then, in conclusion, propose an hypothesis of the mechanism of the action of arsenite.

(1) We have shown that AgBr does react to a measurable extent with monosodium arsenite, or at least goes into solution.

(2) This solution can deposit silver either spontaneously or by addition of a developer. This deposition is accelerated by colloid silver.

(3) Colloidal silver is also coagulated by this solution.

(4) Clark has admitted in answer to Lüppo-Cramer that AgBr goes into solution and probably forms complexes with the more alkaline arsenites than the monosodium. Our own work is in substantial agreement with this.

(5) He likewise agrees that these complexes are unstable and decompose with deposition of silver, even without development.

(6) Clark has shown that physical development of a plate treated with arsenite and followed by fixation gives an image, although silver arsenite is soluble in sodium thiosulphate.

(7) He has stated further that when chromic acid has reduced the sensitivity of a plate, very little if any latent image is produced by arsenite treatment.

(8) Prolonged treatment of a plate with arsenite produces reversal.

We believe the following explanation to be compatible with all of these facts:

First, the arsenite tends more or less according to its alkalinity to dissolve and react with the silver bromide of the plate.

Secondly, this silver salt is probably absorbed *by*, then tends to decompose, and deposit silver possibly *on* the sensitive nuclei.

This tendency to decompose is less where no nuclei are present on the surface, as, for instance, after chromic acid treatment. It is also less with the monosodium salt than with the more alkaline arsenite. There seems to be likewise some variation from emulsion to emulsion, probably due to the variation in number and size of the sensitive nuclei in different emulsions. In any case, where silver has actually been deposited on the sensitive nuclei, developability has been produced to some extent for a given developer.

Thirdly, it is not alone the fact that silver arsenite is formed when the AgBr goes into solution, but likewise its tendency to deposit silver on the sensitive nuclei which causes developability. Therefore, when a plate is treated with arsenite followed by fixation and is then developed in a physical developer, the fixing removes excess silver bromide and silver arsenite, but the latter has already had its chance to deposit some silver and make the nuclei large enough for development by the physical developer.

Fourthly, when the sensitive nuclei on the surface of the grains are removed by chromic acid prior to arsenite treatment, the tendency of the silver arsenite to deposit silver is less than in a plate not treated with chromic acid, especially with the monosodium arsenite treatment, hence one would expect less developability after chromic acid treatment.

Fifthly, reversability is explained for the plates treated with the more alkaline arsenites for a long period on the ground that there is less silver bromide left to develop; the arsenite has dissolved part or most of it. Where there is less AgBr, obviously there will be less silver and hence less light stopping power.

In the case of the monosodium salt, while this explanation may suffice to some extent it needs some further supplementing, and we offer the following as a possibility: Decomposition of AgBr which dissolves, to form Ag arsenite, leaves bromide ions in the solution. These accumulate with increasing time of treatment and are absorbed in increasing quantity by the residual AgBr and are not washed out by the subsequent washing. This prevents as rapid development of those parts as the parts which have been given shorter times of treatment.

We have not, as yet, tested this possibility, but it should not be difficult to determine whether or not Br ions are retained by the latent image or by the residual AgBr as suggested. Clark's

suggestion that arsenite peptizes the silver is not entirely in agreement with our observation that arsenite solutions tend to coagulate colloidal silver.

Clark has used the fact of the action of arsenite, hydrogen peroxide, or other solutions on the photographic plates as an argument against the quantum theory of light exposure, but it is difficult for us to understand his line of reasoning. It appears to us that if anything, the effects produced by these solutions lend support to the Einstein-Silberstein theory of discrete action, since there are few if any to-day who would claim that molecules are not separate entities, discontinuous parts of matter. True, there are many more molecules of arsenite or H_2O_2 in the solutions which have been used to produce the latent image than grains of silver halide on the plate. But from Jones and Schoen's data [15] there is also a great excess of light quanta over the number of grains. Such an excess is necessary when one considers what an exceedingly small target area the sensitive spot presents to the light, or molecules of solute.

Finally, we wish to thank Miss B. M. LaBar for assistance in coating the one-grain-layer plates, and Mr. A. M. Burgess for his assistance in the experimental work with the arsenites.

The Emission Spectra of Mixed Alkali Vapors. F. H. NEWMAN. (*Phil. Mag.*, July, 1924.)—In a previous paper the author reported his discovery of interesting features in the spectrum of the light emitted by an arc formed between two pools of an alloy of potassium and sodium. This spectrum had two continuous parts, one in red, the other in the blue-green region with a fluting spectrum. "In addition to the well-known series of sodium and potassium lines, there were exhibited the bands previously observed. With small vapor pressures they were faint, increasing in brightness as the tube was heated or the current increased. The increased intensity is due to the greater vapor pressure. As the latter was raised, the bands merged into a continuous band which extended from λ6780 to λ5485. In addition, at higher temperatures another band which appeared continuous between the limits λ4750 and λ4040 was exhibited. It is probable that the complete banded spectrum arises from three distinct sets of molecules, *viz.*, potassium, sodium, and molecules consisting of associated atoms of both elements." G. F. S.

[15] L. A. Jones and A. L. Schoen, *J. Opt. Soc. Am.*, **7**, 213 (1923).

HEAVISIDE'S OPERATORS IN ENGINEERING AND PHYSICS.*

BY

ERNST JULIUS BERG, M.E., Sc.D.

Professor of Electrical Engineering, Union University;
Consulting Engineer, General Electric Company,
Schenectady, N. Y.

HEAVISIDE'S three volumes on electromagnetic theory are a compilation of papers and articles covering a period from 1891 to 1912, during which time it seems that he was interested in almost every phase of physics and mathematics.

A very large part of this work is devoted to electrical phenomena and more particularly to the calculation by operational methods of disturbances in electrical networks. Maxwell had already given a most comprehensive theory of almost anything that could happen in electrical systems, but his mathematics, which was conventional mathematics, was with the greatest difficulty only applicable to many practical problems. The task therefore that Heaviside set before himself was to evolve some new point of view which would simplify the treatment without impairing the accuracy attained.

This he succeeded in doing admirably by the introduction of his operational solution and the expansion theorem.

The particular physical problems that he studied led to linear differential equations with constant coefficients and certain types of partial differential equations. In particular, he studied the behavior of systems initially at rest to which forces or impulses were suddenly applied at $t = 0$. Prior to Heaviside's time the use of operators was well established.

So, for instance, it was customary to write the solution of differential equation:

$$\frac{dy}{dt} - ay = f(t) \quad \text{as} \quad y = \frac{1}{D - a} f(t)$$

where D indicated the operation $\frac{d}{dt}$.

* Lectures at the Moore School of Electrical Engineering under the joint auspices of the Moore School and The Franklin Institute, February 28, 29, March 5, 6, 7, 1924.

The solution of this equation is, of course,

$$y = c\varepsilon^{at} + \varepsilon^{at}\int \varepsilon^{-at} f(t)\,dt$$

$c\varepsilon^{at}$ being the complementary function and the second term the particular integral. The first term is independent of $f(t)$ and depends upon the terminal conditions only. The second requires integration, often very laborious, especially in the case of differential equations of higher order.

Heaviside in a way reversed the problem and decided to choose a particular function of (t), $f(t)$, such that integration or the determination of the integration constants would not be necessary. He used symbol p for the time differentiator, that is $p = \frac{d}{dt}$, and, furthermore, stipulated that integration and differentiation should in his system be inverse processes, and that any expressions involving p should follow the laws of ordinary algebra, that is, p could and should be treated as any other constant such as a, b or c, etc.

The particular function of (t) that he chose he denoted by 1, the unit function, and he stipulated (p. 130, Vol. ii) that

$$\frac{1}{p^n}\,1 = \frac{t^n}{\lfloor n} \qquad (1)$$

In other words

$$\frac{1}{p}\,1 = t$$
$$\frac{1}{p^2}\,1 = \frac{t^2}{\lfloor 2}$$
$$\frac{1}{p^n}\,1 = \frac{t^n}{\lfloor n}$$

Since $p = \frac{d}{dt}$, $\frac{1}{p}$ must mean integration if it is to signify the inverse operation. Thus $\frac{1}{p}\,1 = \int 1\,dt$ and since this should be equal to t, it is evident that $\frac{1}{p}\,1 = \int_0^t 1\,dt$ and his unit function is a straight vertical line at $t = 0$, which becomes horizontal from $t = 0$ to $t = \infty$, as shown in Fig. 1.

The shaded area is $t \times 1 = t$ and is $\int_0^t 1\,dt$. Since then $\frac{1}{p}\,1 = t\,1$,*

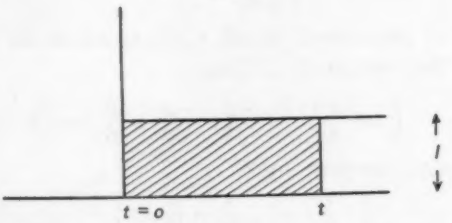
* Equ. (1) is given in the form given by Heaviside. It might also have been written $\frac{1}{p^n}\,1 = \frac{t^n}{\lfloor n}\,1$, since 1 becomes a mere numerical factor when $t > 0$.

it is evident that

$$p.\frac{1}{p}1 = \frac{d}{dt}(t)1 = 1$$

Thus the condition of inverse relation between differentiation and integration is satisfied. Note that p and $\frac{1}{p}$ do not represent ordinary differentiation and integration. They are peculiar and new

FIG. 1.



operators which in *some* ways are similar to differentiation and integration.

So, for instance, $p1$, considered from the point of view of differential calculus, would mean something which at $t = 0$ would rise to infinity and then come down to zero.

To integrate such a thing is, of course, nonsense, but with Heaviside's operator $\frac{1}{p}$, something very definite is obtained. We get back the unit function. This is a very important distinction.

This may be illustrated in the case of charging a condenser when the leads have no resistance. We have then to deal with the "resistance operator" of a condenser which will be shown later to be $\frac{1}{pc}$ or $i = pcE1$.

This equation shows that the current rises to infinity and comes back to zero at $t = 0$ and remains zero forever after. The *charge* taken by the condenser is $\int idt$ in conventional mathematics and with Heaviside's operator it is

$$\frac{i}{p} = \frac{1}{p}pcE1 = cE1.$$

Which means that the charge taken is cE and that it remains cE forever after if nothing happens to the condenser. All of which agrees with facts.

It is easily shown that his operator follows completely the common laws of algebra.

He investigated other operators on the unit function, so, for instance (p. 88, Vol. ii), he showed that

$$\frac{1}{1+\frac{a}{p}}1=\varepsilon^{-at}$$

or since p is to obey all laws of algebra

$$\frac{p}{p+a}1=\varepsilon^{-at}$$

His method of procedure in all such cases is as he calls it to "algebrize" the operator. Then

$$\left(1+\frac{a}{p}\right)^{-1}1=\left(1-\frac{a}{p}+\frac{a^2}{p^2}+\cdots\right)1$$

by the binominal theorem since

$$\frac{1}{p}1=t\ ,\ \frac{1}{p^2}=\frac{t^2}{\lfloor 2}\ \text{etc.}$$

$$\frac{p}{p+a}1=1-at+\frac{a^2t^2}{\lfloor 2}-\frac{a^3t^3}{\lfloor 3}+\cdots=\varepsilon^{-at}\ .^{*}\qquad(2)$$

In a similar way (p. 129, Vol. ii), it may be shown that

$$\frac{1}{p+a}1=\frac{1}{-a}\left[\varepsilon^{-at}-1\right].\qquad(3)$$

It is

$$\frac{1}{p+\alpha}1=\frac{1}{p}\left(\frac{1}{1+\frac{\alpha}{p}}\right)1=\frac{1}{p}\left(1-\frac{\alpha}{p}+\frac{\alpha^2}{p^2}-\frac{\alpha^3}{p^3}+\cdots\right)1$$

$$=\left[\frac{1}{p}-\frac{\alpha}{p^2}+\frac{\alpha^2}{p^3}-\frac{\alpha^3}{p^4}\right]1$$

$$=t-\alpha\frac{t^2}{\lfloor 2}+\alpha^2\frac{t^3}{\lfloor 3}-\cdots$$

$$=\frac{1}{\alpha}\left[\alpha t-\alpha^2\frac{t^2}{\lfloor 2}+\alpha^3\frac{t^3}{\lfloor 3}-\cdots\right]=-\frac{1}{\alpha}\left(\varepsilon^{-at}-1\right).$$

The application of these relations is illustrated in several problems, for instance, in the case of suddenly connecting an inductive circuit to a storage battery.

Previous to Heaviside, we would write

$$e=ir+L\frac{di}{dt}.$$

Since the sudden connection of the battery to the circuit means the application of an e.m.f., which is zero before $t=0$ and a con-

* This really becomes $\epsilon^{-at}1$. It means that ϵ^{-at} is multiplied by the unit function. It is thus zero up to time zero and ϵ^{-at} afterwards.

stant forever after, we are evidently dealing with his unit function. Thus

$$E1 = ir + Lpi$$

or, since p is in his system to be treated as an ordinary symbol,

$$E1 = i(r + pL) \text{ or } i = \frac{E}{r + pL} 1$$

or

$$i = \frac{E}{L\left(p + \frac{r}{L}\right)} 1$$

therefore from equation (3)

$$i = \frac{E}{L}\left[\frac{1}{-\frac{r}{L}}\left(\varepsilon^{-\frac{r}{L}t} - 1\right)\right]$$

or the well-known relation

$$i = \frac{E}{r}\left[1 - \varepsilon^{-\frac{r}{L}t}\right]$$

From this point on to his important expansion theorem there is little clue from Heaviside's writings. But it is not difficult to fill in the steps. It has undoubtedly been done by many from various points of view. A complete list of authors on Heaviside's mathematics is unfortunately not available. The writer can only mention a few names that happen to be familiar to him—Pleijel and Herlitz in Sweden; Carson, Bush, Press, J. J. Smith and Cohen in this country.

The derivation of the expansion theorem has undoubtedly been given by them, yet, since the proof is short, it may be advisable to include it here.

On page 127 (Vol. ii) Heaviside writes his theorem thus:

$$i = \frac{E}{Z_0} + E\Sigma \frac{\varepsilon^{pt}}{p\frac{dZ}{dp}}$$

and this is the form discussed by Cohen. On page 135 (Vol. ii) he writes:

$$i = \frac{Y_0}{Z_0} E + E\Sigma \frac{Y\varepsilon^{pt}}{p\frac{dZ}{dp}} \qquad (4)$$

This latter relation is often preferable and will therefore be proven in this paper.

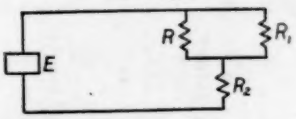
Before entering on the proof, however, it is desirable to discuss the nature and particular value of the operational solution.

Heaviside concluded evidently that the use of differentials and integrals was undesirable—if it was possible to do without them.

FIG. 2.

He also believed that even the most primitive electrical engineer could apply Ohm's law to the permanent condition in a direct current network involving only e.m.f.'s and resistances. In short,

FIG. 3.



he adopted the premise that in the circuit illustrated in Fig. 2 any electrical engineer could show that the joint resistance is

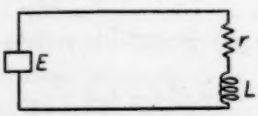
$$R_0 = \frac{RR_1}{R + R_1}$$

thus

$$i = \frac{E}{R_0} = E\frac{(R + R_1)}{RR_1},$$

the steady current after the system has become stable.

FIG. 4.



Again, in the circuit shown in Fig. 3 he assumed it as evident that the total resistance was

$$R_0 = R_2 + \frac{RR_1}{R + R_1}$$

and that the current in any branch could easily be found as function of the steady impressed e.m.f.

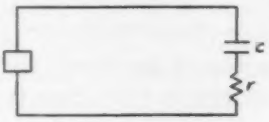
He then considered more complicated circuits as, for instance, that shown in Fig. 4. The instantaneous relation he assumed as known, or, if not, he derived it: It is

$$e = ir + L\frac{di}{dt}$$

Thus under his conditions we write

$$E1 = ir + pLi = i(r + pL)$$

FIG. 5.



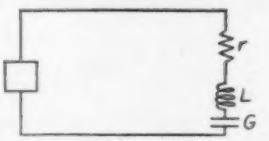
In the case of Fig. 5,

$$e = ir + \frac{1}{c}\int idt$$

Thus under his conditions,

$$E1 = ir + \frac{i}{pc} = i\left(r + \frac{1}{pc}\right)$$

FIG. 6.



In the case of Fig. 6,

$$e = ir + L\frac{di}{dt} + \frac{1}{c}\int idt$$

Thus under his conditions,

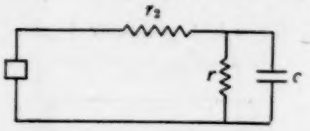
$$E1 = ir + pLi + \frac{i}{cp} = i\left(r + pL + \frac{1}{pc}\right)$$

His so-called "resistance operator" for a circuit of resistance and inductance in series is $z = r + pL$. Note that this does not mean that the impedance is $r + pL$. It merely means that if z operates on unit function of E, the current under Heaviside's condition can be found. The resistance operator for a circuit of resistance and capacity in series is $z = r + \frac{1}{pc}$.

The operator in the case of resistance, inductance and capacity in series is $z = r + pL + \frac{1}{pc}$.

Consider next the circuit shown in Fig. 7, which might represent a leaky condenser in series with a resistance. We may pro-

FIG. 7.



ceed to set up the equation of the steady current as if the various branches contained resistances only, that is, from Fig. 3 we get

$$P_0 = R_2 + \frac{RR_1}{R + R_1}$$

$$\therefore z_0 = r_2 + \frac{r\frac{1}{pc}}{r + \frac{1}{pc}} = r_2 + \frac{r}{1 + rpc} = \frac{r_2 + rr_2pc + r}{1 + rpc}$$

$$\therefore i = \frac{E1}{z_0} = E\frac{(1 + rpc)}{r + r_2 + rr_2pc}1 \qquad (a)$$

This relation he calls the *operational solution* and writes it:

$$i = E\frac{Y}{Z}1 \qquad (5)$$

Thus in this case

$$Y = 1 + pcr$$
$$Z = r + r_2 + rr_2pc$$

In connection with this it might be asked why not write the operational solution

$$i = E\frac{\frac{1 + pcr}{r + r_2 + rr_2pc}}{1}1 \qquad (b)$$

in which case Y would be $\frac{1 + pcr}{r + r_2 + rr_2pc}$ and $Z = 1$, or why not write:

$$i = E\frac{1}{\frac{r + r_2 + rr_2pc}{1 + pcr}}1 \qquad (c)$$

in which case Y would be unity and

$$Z = \frac{r + r_2 + rr_2pc}{1 + pcr}?$$

It will be clear from what follows in the proof of the expansion theorem that equations (a) and (c) are equally applicable, but that equation (b) would not do.

The rule is that the operational solution must be of such form that when Z, the denominator, is equated to zero we obtain the largest number of roots of p. In this case $Z = 0$ gives only one root, $p = -\frac{r + r_2}{rr_2C}$ whether we use (a) or (c), but equation (b) gives no roots at all. In most problems Z contains higher powers of p than the first when, of course, several roots are obtained. From this discussion it should be evident how the operational solution is obtained at least in networks of concentrated resistance, inductance and capacity.

A number of examples will be given later which involve not only resistance, inductance and capacity, but also mutual inductances. These will help to clear up any doubts that may exist at this time.

Referring now to equation (5),

$$i = E\frac{Y_{(p)}}{Z_{(p)}}1$$

Z always contains p in various powers. Thus, in general, $Z_{(p)}$ can be written:

$$Z_{(p)} = p^n + ap^{n-1} + \cdots\cdots$$

Let the roots corresponding to $Z_{(p)} = 0$ be p_1, p_2, p_3, etc.,

$$\therefore Z_{(p)} = (p - p_1)(p - p_2)(p - p_3)\cdots\cdot$$

$$\therefore \frac{Y_{(p)}}{Z_{(p)}} = \frac{Y_{(p)}}{(p - p_1)(p - p_2)(p - p_3)\cdots\cdot}$$

When this expression is broken into partial fractions and all roots are different and none is zero, we get:

$$\frac{Y_{(p)}}{Z_{(p)}} = \frac{A}{p - p_1} + \frac{B}{p - p_2} + \frac{C}{p - p_3} + \cdots\cdot \qquad (6)$$

Where

$$A = \frac{Y_{(p_1)}}{Z'_{(p_1)}},\ B = \frac{Y_{(p_2)}}{Z'_{(p_2)}},\ \text{etc.} \qquad (7)$$

Where Z' is $\frac{d}{dp}Z$ and $Z'_{(p_1)}$ is the particular value of Z' when p_1 is substituted for p. For the explanation of this relation see,

for instance, Todhunter's "Integral Calculus" (second chapter) or Pierce's "Table of Integrals" (p. 14).

We get also

$$\frac{Y_{(p)}}{Z_{(p)}}1 = A\frac{1}{p-p_1}1 + B\frac{1}{p-p_2}1 + \cdots \qquad (8)$$

Substituting the value for $\frac{1}{p-p_1}1$ from (3), it is easily seen that equation (8) becomes

$$\frac{Y_{(p)}}{Z_{(p)}}1 = -\left[\frac{Y_{(p_1)}}{p_1 Z'_{(p_1)}} + \frac{Y_{(p_2)}}{p_2 Z'_{(p_2)}} + \cdots\right] + \frac{\varepsilon^{p_1 t}Y_{(p_1)}}{p_1 Z'_{(p_1)}} + \frac{\varepsilon^{p_2 t}Y_{(p_2)}}{p_2 Z'_{(p_2)}} + \cdots$$

The parenthesis may be written

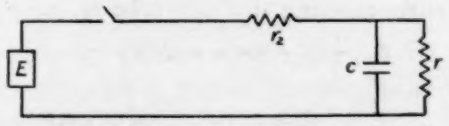
$$+\left[\frac{A}{-p_1} + \frac{B}{-p_2} + \cdots\right]$$

and this is nothing but $\frac{Y_{(o)}}{Z_{(o)}}$, as seen from equation (6) when $p = o$.

$$\therefore i = E\left[\frac{Y_{(o)}}{Z_{(o)}} + \sum_{p_1 p_2 p_3 \cdots} \frac{Y_{(p)}\varepsilon^{pt}}{p\frac{dz}{dp}}\right] \qquad (9)$$

which is the same equation as Heaviside gives on page 135 (Vol. ii). Note, however, that in this demonstration it has been

FIG. 8.



assumed that all roots of $Z_{(p)} = o$ are unequal. The method to be used when several roots are equal, or when one root is zero, will be given later. (See page 663.)

To illustrate the application of this theorem to the problem cited above when suddenly a leaky condenser in series with a resistance is connected to a storage battery of potential difference E, we have $Z_{(p)} = r + r_2 + rr_2pc$. (See equation (a).)

$$\therefore Z_{(p)} = o \text{ gives } p = -\frac{r+r_2}{rr_2c} \therefore p_1 = -\frac{r+r_2}{rr_2c}$$

$$\frac{dz}{dp} = rr_2c \quad \therefore \quad p\frac{dz}{dp} = prr_2c$$

and

$$p\frac{dz}{dp} \text{ for } p = p_1 = -\frac{r+r_2}{rr_2c} \times rr_2c = -(r+r_2)$$

$$Y_{(p_1)} = 1 - \frac{r+r_2}{rr_2c} \times rc = 1 - \frac{r}{r_2} - 1 = -\frac{r}{r_2}$$

$$\frac{Y_{(0)}}{Z_{(0)}} = \frac{1}{r+r_2}$$

$$\therefore i = E\left[\frac{1}{r+r_2} + \frac{r}{r_2(r+r_2)}\varepsilon^{-\frac{r+r_2}{rr_2c}t}\right]$$

For $t=0$, $i=\frac{E}{r_2}$ and for $t=\infty$, $i=\frac{E}{r+r_2}$.

It is of interest to note that the initial and final currents can always be obtained directly from the operational solution.

In this case

$$i = E\frac{(1+pcr)}{r+r_2+pcrr_2}1 \qquad (a)$$

When we remember the shape of the unit function, we realize that for $t=0$, $p=\frac{d}{dt}=\infty$; for $t=\infty$, $p=0$.

Therefore, for $t=0$ when $p=\infty$ or is very large compared with any finite quantity, we get

$$i = \frac{Epcr}{pcrr_2}1 = \frac{E}{r_2}$$

For $t=\infty$, $p=0$, and therefore $i=E\frac{1}{r+r_2}$.*

---

* The operational solution is the solution of the permanent alternating current condition when vector representation is used if for p is substituted $j\omega$, thus $p^2=-\omega^2$, $p^3=-j\omega^3$, etc.

Heaviside brought out this fact in 1892, or perhaps earlier; it had also been shown by Kennelly in 1892.

The proof is simple when it is recollected that a convenient equation of a revolving vector is:

$$\dot{I} = I\varepsilon^{j\omega t} \text{ thus } \frac{d}{dt}\dot{I} = p\dot{I} = j\omega I\,\varepsilon^{j\omega t} = j\omega\dot{I}$$

Referring then to equation (9),

$$\dot{I} = E\frac{(1+j\omega rc)}{r+r_2+j\omega crr_2}$$

$$= E\frac{r+r_2+\frac{r^2}{x_c^2}r_2+j\frac{r^2}{x_c}}{(r+r_2)^2+\frac{r^2}{x_c^2}r_2^2}$$

where $x_c=\frac{1}{\omega c}$.

The operational equation brings out some other interesting features. Since for $t=0$, $p=\infty$ and for $t=\infty$, $p=0$, it is evident that if the denominator is of a higher power of p than the numerator, the initial current must be zero.

If numerator and denominator are of the same power of p, the initial current is of a definite value, not zero.

If the numerator is of higher power of p than the denominator, then the initial rush of current is infinite, but after an infinitely short time it settles down to a definite value.

It might be well to say here that this condition can never be met in any actual problem, whether mechanical or electrical. It would exist in the case, for instance, where a condenser is suddenly connected to a battery with wires of no resistance, which obviously is impossible.

However, even then the method throws some light on the subject. We would obviously expect to find an instantaneous infinite rush of current during an exceedingly short time such that $\int idt$ would be the charge which corresponds to the voltage impressed.

The operational solution obviously is:

$$i = E\frac{1}{\frac{1}{pc}}1 = pcE1$$

which means an instantaneous rush of current of infinite magnitude and the charge $=\frac{1}{p}=cE$, which is the proper charge.

ADDITIONAL OPERATORS EMPLOYED WHEN A NETWORK IS SUDDENLY CONNECTED TO AN ALTERNATOR, INSTEAD OF TO A BATTERY OF CONSTANT VOLTAGE.

Referring to equation (2),

$$\frac{p}{p-a}1 = \varepsilon^{at}.$$

$$\sin \omega t = \frac{\varepsilon^{j\omega t}-\varepsilon^{-j\omega t}}{2j} = \frac{1}{2j}\left[\frac{p}{p-j\omega}1 - \frac{p}{p+j\omega}1\right]$$

$$= \frac{p}{2j}\left[\frac{p+j\omega-p+j\omega}{p^2+\omega^2}\right]1 = \frac{p\omega}{p^2+\omega^2}1. \qquad (10)$$

Similarly

$$\cos \omega t = \frac{p^2}{p^2 + \omega^2} 1 \cdot \tag{11}$$

$$\therefore \sin (\omega t \pm \varphi) = \sin \omega t \cos \varphi \pm \cos \omega t \sin \varphi$$

$$= \frac{p\omega \cos \varphi \pm p^2 \sin \varphi}{p^2 + \omega^2} 1 \tag{12}$$

and

$$\cos (\omega t \pm \varphi) = \frac{p^2 \cos \varphi \mp p\omega \sin \varphi}{p^2 + \omega^2} 1 \cdot * \tag{13}$$

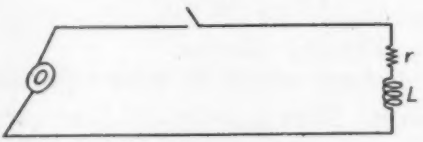
This relation, I believe, was first shown by Pleijel. Similarly

$$\epsilon^{\beta t} \sin (\omega t \pm \varphi) = \frac{p\omega \cos \varphi \pm p (p + \beta) \sin \varphi}{(p + \beta)^2 + \omega^2} 1 \tag{14}$$

$$\epsilon^{\beta t} \cos (\omega t \pm \varphi) = \frac{p (p + \beta) \cos \varphi \mp p\omega \sin \varphi}{(p + \beta)^2 + \omega^2} 1 \cdot \tag{15}$$

The application of the additional operator will be shown in

FIG. 9.



case of an alternator being connected to an inductive circuit as is shown in Fig. 9.

Let the voltage generated be a simple sine wave and the switch closed at $t = 0$ when the value of the impressed e.m.f. is $E \sin \varphi$.

The equation for the impressed e.m.f. is then $e = E \sin (\omega t + \varphi)$.

The procedure is then to write first the operational solution as if unit e.m.f. was impressed. It is: $i = \frac{E}{r + pL} 1$. Then introduce the additional operator which converts the sine wave to a wave of unit function.

* The use of Duhamel's integral, which is often very convenient in calculating problems involving periodic forces, has not been illustrated in this paper. From one point of view this is to be regretted, because certainly in many problems, particularly those involving "distributed" constants, this integral is more convenient than the method given. On the other hand, it is felt that there is a decided advantage to show that solutions involving such forces can always be obtained in the strictly operational way.

Thus the operational solution becomes

$$i = \frac{E}{r + pL} \cdot \frac{\omega p \cos\varphi + p^2 \sin\varphi}{\omega^2 + p^2}, \tag{16}$$

$$\therefore Y_{(p)} = \omega p \cos\varphi + p^2 \sin\varphi$$

and

$$Z_{(p)} = (r + pL)(\omega^2 + p^2)$$

The solution of equation (16) gives the instantaneous values of the current. We note that the denominator is of a higher power of p than the numerator, thus the initial current value is zero. But in this case we cannot say that for $t = \infty$, $p = 0$, it is not zero because the final current is obviously not steady. If we desire to know the final value without solving the equation by the expansion theorem, we may, however, resort to the vector representation as discussed previously and write

when $p = j\omega$

$$\dot{I} = \frac{\dot{E}}{r + pL}$$

$$\therefore \dot{I} = \frac{\dot{E}}{r + j\omega L}$$

which is the well-known solution.

It is now perfectly simple to solve equation (16) by the expansion theorem. Since $z_{(p)} = (r + pL)(\omega^2 + p^2)$, we get three roots, $p_1 = -\frac{r}{L}$, $p_2 = +j\omega$, $p_3 = -j\omega$. We proceed then to find $d\frac{z_{(p)}}{dp}$ and then $p\frac{dz_{(p)}}{dp}$, for $p = p_1$, p_2 and p_3, etc.

The result will be three terms ($\frac{Y_{(o)}}{z_{(o)}}$ is easily seen to be zero in this case):

$$Ae^{-\frac{r}{L}t} + Be^{j\omega t} + Ce^{-j\omega t}$$

The last two terms will combine to a pure sine wave without decrement and are, therefore, the permanent condition, which usually is well known to engineers.

In this case the permanent a. c. current is, of course,

$$i_p = \frac{E}{Z}\sin(\omega t + \varphi - \alpha)$$

So that it is only necessary to calculate the value of i from the expansion theorem for the root $p = p_1 = -\frac{r}{L}$, and then to add i_p to the solution.

However, the solution is not always as easily gotten as that.

We will therefore consider a general simplification of work when the denominator $Z_{(p)}$ is in the form of a product.

Let then $$\frac{Y_{(p)}}{Z_{(p)}} = \frac{Y_{(p)}}{h_{1(p)} \cdot h_{2(p)}}$$

$$\therefore Z'_{(p)} = \frac{dz_{(p)}}{dp} = h_{1(p)}\, h_2'_{(p)} + h_{2(p)}\, h_1'_{(p)}$$

and $$pZ'_{(p)} = p\,[h_{1(p)}\, h_2'_{(p)} + h_{2(p)}\, h_1'_{(p)}]$$

When in this expression we substitute the root or roots obtained when $h_{1(p)} = 0$, we get only the second term left since $h_{1(p)}$ obviously becomes zero. A similar argument applies in regard to the second set of roots, so that the net result is, for instance, in case $h_{1(p)}$ has only one root, p_1, and $h_{2(p)}$ two roots, p_2 and p_3,

$$i = E\left[\frac{Y_{(0)}}{Z_{(0)}} + \frac{Y_{(p)}\varepsilon^{pt}}{ph_{2(p)}h_1'_{(p)}} \text{ for } p = p_1 + \frac{Y_{(p)}\varepsilon^{pt}}{ph_{1(p)}h_2'_{(p)}} \text{ for } p = p_2 + \frac{Y_{(p)}\varepsilon^{pt}}{ph_{1(p)}h_2'_{(p)}} \text{ for } p = p_3\right]$$

This relation, I believe, has first been worked out by Pleijel or Herlitz.

In the particular problem given above,

$$h_{1(p)} = r + pL \therefore h_1'_{(p)} = L \text{ and } ph_1'_{(p)} = pL$$

$$h_{2(p)} = \omega^2 + p^2 \therefore h_2'_{(p)} = 2p \text{ and } ph_2'_{(p)} = 2p^2$$

Thus

$$ph_{2(p)}\, h_1'_{(p)} = pL\,(\omega^2 + p^2) \text{ and } ph_{1(p)}\, h_2'_{(p)} = 2p^2\,(r + pL)$$

$$ph_{2(p)}\, h_1'_{(p)} \text{ for } p = p_1 \text{ becomes } -\frac{r}{L^2}z^2 \text{ where } z^2 = r^2 + \omega^2 z^2 = r^2 + x^2$$

$$Y_{(p)} \text{ for } p = p_1 \text{ becomes } z\frac{r}{L^2}\sin(\varphi - \alpha) \text{ where } \tan\alpha = \frac{x}{r}$$

Thus the first term becomes:

$$-\frac{E}{Z}\varepsilon^{-\frac{r}{L}t}\sin(\varphi - \alpha)$$

$$ph_{1(p)}\, h_2'_{(p)} \text{ for } p = p_2 = j\omega \text{ becomes } -2\omega^2\,(r + j\omega L)$$

$$Y_{(p)} \text{ for } p = p_2 = \omega^2\,(j\cos\varphi - \sin\varphi)$$

$$\therefore \frac{Y_{(p)}}{ph_{1(p)}\, h_2'_{(p)}} \text{ for } p = p_2 = \frac{j\cos\varphi - \sin\varphi}{-2\,(r + j\omega L)} = \frac{\cos\varphi + j\sin\varphi}{2j\,(r + j\omega L)}$$

$$= \frac{\cos(\varphi - \alpha) + j\sin(\varphi - \alpha)}{2jz}.$$

Similarly

$$\frac{Y_{(p)}}{ph_{1(p)}\,h_2'_{(p)}} \text{ for } p = p_2 = \frac{j\cos\varphi + \sin\varphi}{2(r - j\omega L)} = \frac{-\cos(\varphi - \alpha) + j\sin(\varphi - \alpha)}{2jz}.$$

Thus the last two terms become:

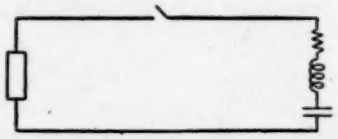
$$\frac{E}{Z}\left[\cos(\varphi - \alpha)\,\frac{\varepsilon^{j\omega t} - \varepsilon^{-j\omega t}}{2j} + \sin(\varphi - \alpha)\,\frac{\varepsilon^{j\omega t} + \varepsilon^{-j\omega t}}{2}\right] = \frac{E}{Z}\sin(\omega t + \varphi - \alpha)$$

which is the permanent value and the solution is:

$$i = \frac{E}{Z}\left[\sin(\omega t + \varphi - \alpha) - \varepsilon^{-\frac{r}{L}t}\sin(\varphi - \alpha)\right]$$

The process is somewhat lengthy, but as stated above, when it is once done, it is known that the roots $\pm j\omega$ give the permanent

FIG. 10.



condition, so that it is really only necessary to calculate the condition for the other root or roots, if the permanent condition is known. If not, it is simplest to obtain it first in vector form by substituting $j\omega$ in the operational solution $i = \frac{E1}{r + pL}$ in this particular case.

In order to show the procedure when all roots are not equal, the circuit illustrated in Fig. 10 will be considered. It is evident that:

$$i = \frac{E}{r + pL + \frac{1}{pc}}1 \qquad (17)$$

If the equation is left in its present form

$$Y_{(p)} = 1$$

$$Z_{(p)} = r + pL + \frac{1}{pc}$$

Thus $Z_{(p)} = 0$ gives

$$\left.\begin{aligned} p_1 &= -\alpha + j\omega \\ p_2 &= -\alpha - j\omega \end{aligned}\right\} \text{ when } \frac{1}{cL} > \frac{r^2}{4L^2}$$

Where

$$\alpha = \frac{r}{2L}, \omega = \sqrt{\frac{1}{cL} - \alpha^2} = \sqrt{\omega_o^2 - \alpha^2}$$

$$\therefore \alpha^2 + \omega^2 = \omega_o^2 \text{ if } \omega_o = \sqrt{\frac{1}{Lc}}$$

$$d\frac{z_{(p)}}{p} = L - \frac{1}{p^2 c} \text{ and } \therefore p\frac{dz}{dp} = pL - \frac{1}{pc} = p^2\frac{Lc-1}{pc}$$

$p\frac{dz}{dp}$ for $p = p_1$ then becomes $+ 2j\omega L$

and

$p\frac{dz}{dp}$ for $p = p_2$ becomes $- 2j\omega L$

$$\frac{Y_{(o)}}{z_{(o)}} = o$$

$$\therefore i = E\left[\frac{\varepsilon^{(-\alpha+j\omega)t}}{2j\omega L} - \frac{\varepsilon^{(-\alpha-j\omega)t}}{2j\omega L}\right] = \frac{E}{\omega L}\varepsilon^{-\alpha t}\left[\frac{\varepsilon^{j\omega t} - \varepsilon^{-j\omega t}}{2j}\right]$$

$$= E\frac{\varepsilon^{-\alpha t}}{\omega L}\sin \omega t$$

It is readily proven that if $\frac{1}{cL} < \frac{r^2}{4L^3}$, then

$$i = E\frac{\varepsilon^{-\alpha t}}{L\beta}\sinh \beta t$$

where

$$\beta = \sqrt{\frac{r^2}{4L^2} - \frac{1}{cL}}$$

The case of equal roots, that is, when $\frac{1}{cL} = \frac{r^2}{4L^2}$. As these approach equality, ω approaches zero, thus sin ωt approaches ωt

$$\therefore i = E\frac{\varepsilon^{-\alpha t}}{\omega L}\omega t = Et\frac{\varepsilon^{-\alpha t}}{L}$$

But we could have obtained the same relation in another way, equation (17) might also be written

$$i = \frac{pcE}{rpc + p^2cL + 1}1$$

or $$i = \frac{pcE}{cL\left(p^2 + \frac{r}{L}p + \frac{1}{cL}\right)}1 = \frac{pE}{L\left(p^2 + \frac{r}{L}p + \frac{1}{cL}\right)}1 = \frac{pE}{L(p+\alpha)^2}1$$

The question then is, what is the meaning of

$$\frac{E}{L}\frac{p}{(p+\alpha)^2}1 ?$$

Note that

$$\frac{d}{d\alpha}\frac{p}{p+\alpha} = -\frac{p}{(p+\alpha)^2}$$

but $$\frac{p}{p+\alpha}1 = \varepsilon^{-\alpha t}, \text{ and } \therefore \frac{d}{d\alpha}\frac{p}{p+\alpha}1 = -t\varepsilon^{-\alpha t}$$

$$\therefore \frac{p}{(p+\alpha)^2}1 = t\varepsilon^{-\alpha t}$$

and $$i = \frac{E}{L}t\varepsilon^{-\alpha t}*$$

It is, of course, realized that while in these demonstrations the current has always been calculated, the method is in no way limited to that particular variable. Suppose, for instance, that it was desired to find the voltage across the condenser, or more explicitly, the voltage consumed by the condenser after the switch is closed, then since $e_c = \frac{i}{pc}$,

$$e_c = \frac{E}{\left(r + pc + \frac{1}{pc}\right)pc}1 \qquad (18)$$

In the example calculated above which gave a pair of conjugate roots the calculation was not very laborious. This was largely due to the fact that $Y_{(p)}$ was unity. This is, of course, not always the case.

In general the operational equation may be written

$$i = E\frac{Y_{(p)}}{z_{(p)}}1 = E\cdot\frac{Y_{(p)}}{p^2+a_1p+b}1$$

which under certain conditions gives two roots, $-\alpha \pm j\omega$.

It can easily be proven that when the roots of $z_{(p)} = 0$ are conjugates, $p\frac{dz}{dp}$, for $p = p_1$ and for $p = p_2$, must also be conjugate, and so also the two values of $Y_{(p)}$. And finally the two expressions of $\frac{Y_{(p)}}{p\frac{dz}{dp}}$ are conjugates.

---

* When the operational equation is in the form $i = E\frac{Y_{(p)}}{p(p-\alpha)(p-\beta)\cdots}1$ one root is zero. In this case, if it is best to determine the value of $i_1 = E\frac{Y_{(p)}}{(p-\alpha)(p-\beta)\cdots}1$ and later to find i by integration, it is $i = \int_0^t i_1 dt$.

Let the complex expression of $\dfrac{Y_{(p)}}{p\dfrac{dz}{dp}}$ for $p = -\alpha + j\omega$ be $A = a + ja_1$, or $A = A\varepsilon^{j\delta}$, where $A = \sqrt{a^2 + a_1^2}$ and $\tan\delta = \frac{a_1}{a}$. Then the corresponding relation for $p = p_2 = -\alpha - j\omega$ is $B = a - ja_1$, or $B = A\varepsilon^{-j\delta}$.

Therefore the two terms under the summation sign become

$$A\varepsilon^{j\delta}\varepsilon^{(-\alpha+j\omega)t} + A\varepsilon^{-j\delta}\varepsilon^{(-\alpha-j\omega)t}$$

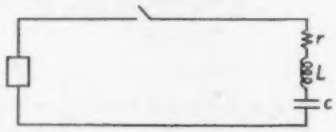
$$= A\varepsilon^{-\alpha t}\left[\varepsilon^{j(\omega t+\delta)} + \varepsilon^{-j(\omega t+\delta)}\right]$$

$$= 2A\varepsilon^{-\alpha t}\cos(\omega t + \delta)$$

$$\therefore i = E\left[\frac{Y_{(\omega)}}{z_{(\omega)}} + 2A\varepsilon^{-\alpha t}\cos(\omega t + \delta)\right] \qquad (19)$$

This method will be demonstrated below.

FIG. 11.



Find the voltage consumed by the inductance L after the switch is closed.

We have $i = \dfrac{E\,1}{r + pL + \frac{1}{pc}}$

$$\therefore e_l = pLi = \frac{pLE}{r + pL + \frac{1}{pc}}\,1 = \frac{p^2E\,1}{p^2 + p\frac{r}{L} + \omega_0^2}$$

$$= \frac{p^2E}{p^2 + 2\alpha p + \omega_0^2}\,1$$

$z = 0$ gives $p = -\alpha \pm j\omega$ where $\omega = \sqrt{\omega_0^2 - \alpha^2}$

$$\frac{dz}{dp} = 2(p + \alpha) \therefore p\frac{dz}{dp} = 2p(p + \alpha)$$

and

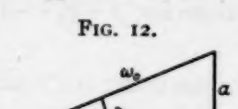
$$\frac{Y_{(p)}}{p\dfrac{dz}{dp}} = \frac{p}{2(p + \alpha)}.$$

For $p = -\alpha + j\omega$ this becomes

$$\frac{-\alpha + j\omega}{2(-\alpha + j\omega + \alpha)} = \frac{-\alpha + j\omega}{2j\omega} = \frac{j\alpha + \omega}{2\omega}$$

$$\therefore \dot{A} = \frac{\alpha^2 + \omega^2}{2\omega} \underline{|\delta} \text{ where } \tan\delta = \frac{\alpha}{\omega}$$

FIG. 12.



or

$$\dot{A} = \frac{\omega_o}{2\omega} \varepsilon^{j\delta}$$

$$\therefore e_i = E\left[\frac{Y_{(o)}}{Z_{(o)}} + \frac{\omega_o}{\omega} \varepsilon^{-\alpha t} \cos(\omega t + \delta)\right]$$

but

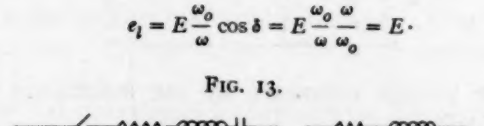
$$\frac{Y_{(o)}}{z_{(o)}} = 0$$

$$\therefore e_i = E\frac{\omega_o}{\omega} \varepsilon^{-\alpha t} \cos(\omega t + \delta) \cdot$$

For $t = 0$,

$$e_i = E\frac{\omega_o}{\omega} \cos\delta = E\frac{\omega_o}{\omega}\frac{\omega}{\omega_o} = E \cdot$$

FIG. 13.



PROBLEM INVOLVING MUTUAL INDUCTANCE.

The conventional relation is, of course,

$$e_1 = i_1 r_1 + L_1\frac{di_1}{dt} + \frac{1}{c_1}\int i_1 dt + M\frac{di_2}{dt}$$

$$0 = i_2 r_2 + L_2\frac{di_2}{dt} + \frac{1}{c_2}\int i_2 dt + M\frac{di_1}{dt} \cdot$$

Thus under Heaviside conditions

$$E1 = i_1\left(r_1 + pL_1 + \frac{1}{pc_1}\right) + i_2 pM = i_1 a_1 + i_2 b$$

$$0 = i_2\left(r_2 + pL_2 + \frac{1}{pc_2}\right) + i_1 pM = i_2 a_2 + i_1 b.$$

From the latter equation $i_2 = -\frac{i_1 b}{a_2}$

$$\therefore E1 = i_1 a_1 - i_1 \frac{b^2}{a_2} = i_1 \frac{(a_1 a_2 - b^2)}{a_2}$$

$$\therefore i_1 = \frac{E a_2}{a_1 a_2 - b^2} 1$$

and

$$i_2 = -\frac{E b}{a_1 a_2 - b^2} 1$$

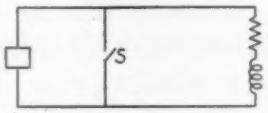
THE EFFECT OF A SHORT CIRCUIT IN A NETWORK.

The initial condition can be determined from the expansion theorem, for instance,

$$i = E\left[\frac{Y_{(o)}}{z_{(o)}} + \Sigma \frac{Y_{(p)}\, \varepsilon^{pt}}{p \frac{dz}{dp}}\right]$$

After the system has reached steady state, this equation obviously becomes $i = E\frac{Y_{(o)}}{z_{(o)}}$. After the short circuit the impressed voltage is zero. Thus we can conclude that the current after the short

FIG. 14.



circuit is the sum of two currents—one, the original, which is $E\frac{Y_{(o)}}{z_{(o)}}$, the other a current due to $-E1$ applied at time $t = 0$.

$$\therefore i = E\frac{Y_{(o)}}{z_{(o)}} - E\left[\frac{Y_{(o)}}{z_{(o)}} + \Sigma\right]$$

or

$$i = -E\Sigma \frac{Y_{(p)}\, \varepsilon^{pt}}{p \frac{dz}{dp}}$$

To illustrate the application, refer to Fig. 14. Switch s is closed at $t = 0$ after the current has become steady, find the current afterwards.

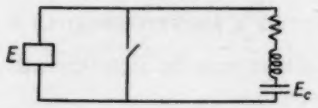
$$i = -\frac{E}{r + pL} 1$$

where the transient term alone is used after the expansion theorem has been applied.

Another example. Find the discharge current of a condenser initially charged to a potential difference E_1 from a battery of voltage E.

$$i = -\frac{E\,1}{r + pL + \frac{1}{pc}} = \frac{E_1\,1}{r + pL + \frac{1}{pc}}.$$

FIG. 15.



Find the voltage across the condenser after the discharge has begun.

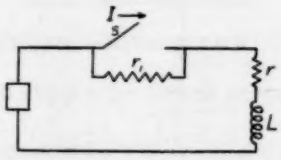
$$e_c = \frac{i}{pc} = \frac{E_1}{pc\left(r + pL + \frac{1}{pc}\right)}\,1$$

where the transient term alone is used after the solution from the expansion theorem has been found.

THE EFFECT OF SUDDENLY CHANGING THE CIRCUIT CONSTANTS.

Refer to Fig. 16 and assume that the circuit has reached its permanent condition before switch s is opened. Before the switch

FIG. 16.



is opened a steady current I flows through the switch. After the switch is opened zero current flows through the switch.

If, therefore, we substitute for the mechanical opening of the switch a certain peculiar e.m.f. in the switch, which at all times after $t = 0$ gives a steady current $-I$, we have accomplished the result.

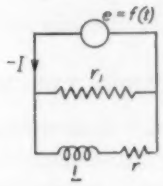
Fig. 16 might advantageously be redrawn, as shown in Fig. 17.

The current caused by the peculiar e.m.f. divides in the two circuits in parallel, so that the current i_1 in r_1 and the total current $-I$ are related as the admittances.

$$\therefore \frac{i_1}{-I} = \frac{\frac{1}{r_1}}{\frac{1}{r_1} + \frac{1}{r + pL}} \quad \therefore i_1 = \frac{r + pL}{r + r_1 + pL}(-I)\,1$$

Note that unit function 1 is inserted. This is evident because the original current in the switch was steady and was $+I$. In order that the current shall be zero, $-I$ must have unit shape.

FIG. 17.



In this particular case the initial current $I = \frac{E}{r}$, so that the current flowing in the circuit is

$$i_1 = -\frac{E}{r}\frac{r + pL}{r + r_1 + pL}1$$

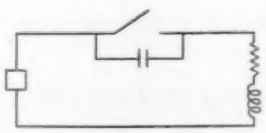
For $t = 0$, $\qquad i_1 = -\frac{E}{r}$,

the minus sign showing that the current in r_1 is in opposite direction to the current I, thus forms the return current after the switch is opened.

The solution of the equation is easily found to be

$$i_1 = -\frac{I}{r + r_1}\left[r + r_1 \varepsilon^{-\frac{r + r_1}{L}t}\right].$$

FIG. 18.



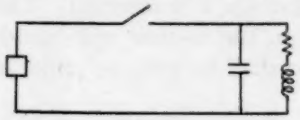
If the switch had been shunted by a condenser (Fig. 18), the following relation is at once apparent:

$$\frac{i_c}{-I} = \frac{pc}{pc + \frac{1}{r + pL}}$$

$$\therefore i_c = -\frac{E}{r}\frac{pc\,(r + pL)}{rpc + p^2cL + 1}1.$$

An interesting problem is the so-called Buzzer excitation used with wave metres (Fig. 19). Here the equation of the current

FIG. 19.



after the buzzer has opened the circuit is obviously the same as in the above case.

HEAVISIDE'S "SHIFTING."

When voltages of certain decrements are impressed on a circuit, for instance, when the short-circuit current of an alternator is calculated, we meet with equations similar to that given below:

$$E\varepsilon^{-\beta t}\sin(\omega t+\varphi) = ir + L\frac{di}{dt}$$

The operational solution of this follows from equation (14). It is:

$$i = E\frac{p\omega\cos\varphi + p(p+\beta)\sin\varphi}{[(p+\beta)^2+\omega^2](r+pL)}1 \qquad (20)$$

Heaviside shows that a correct result is obtained from a simpler equation in which $\varepsilon^{-\beta t}$ is shifted outside of the operational equation so that the equation becomes

$$i = E\varepsilon^{-\beta t}\Big[\underbrace{\frac{p\omega\cos\varphi + p^2\sin\varphi}{p^2+\omega^2}}_{A}\cdot\underbrace{\frac{1}{r+(p-\beta)L}}_{B}\Big]1$$

Note that A simply transforms a simple sin wave, $\sin(\omega t+\varphi)$, to the unit function and B is the operator $\frac{1}{r+pL}$ when for p we have substituted $p-\beta$.

Heaviside writes it on page 294 (Vol. ii),

$$f(p)\,(u\varepsilon^{at}) = \varepsilon^{at}[f(p+a)\,u]$$

where u is a function of t. In this case $\sin(\omega t+\varphi)$. The operator $f(p)$ in this case is $\frac{1}{r+pL}$ and $f(p+a) = \frac{1}{r+L(p-\beta)}$.

Proof: Let first $f(p)=p$; then

$$p\,(u\varepsilon^{at}) = au\varepsilon^{at} + \varepsilon^{at}\frac{du}{dt} = \varepsilon^{at}u(a+p)$$

or in general,

$$p^n (u\varepsilon^{at}) = \varepsilon^{at} u (a+p)^n$$

The relation thus holds when $f(p) = p$. It will obviously hold for any function of (p) as long as it can be expanded in a power series in p.

Incidentally the solution of the problem can be obtained without much work. B may be written

$$\frac{1}{r - \beta L + pL} = \frac{1}{r_o + pL}$$

where $r_o = r - \beta L$.

A gives two roots, $\pm j\omega$, and these it has been shown give the permanent term. Thus the part of the expression under the bracket which corresponds to roots $\pm j\omega$ gives

$$i_1 = \frac{E}{\sqrt{r_o^2 + x^2}} \sin(\omega t + \varphi - \delta)$$

where

$$\tan \delta = \frac{\omega L}{r_o}.$$

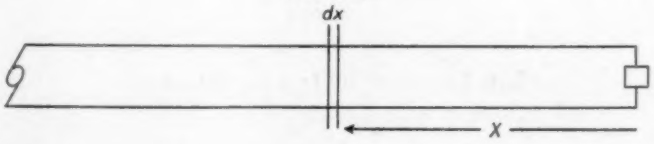
The transient term corresponding to the part of the expression under the bracket is best obtained by resorting to $h_{1(p)}$ and $h_{2(p)}$, as discussed previously, and becomes

$$i_2 = -\frac{E}{\sqrt{r_o^2 + x^2}} \varepsilon^{-\frac{r_o}{L}t} \sin(\varphi - \delta) = -\frac{E}{\sqrt{r_o^2 + x^2}} \varepsilon^{-\left(\frac{r}{L} + \beta\right)t} \sin(\varphi - \delta)$$

Thus

$$i = \frac{E}{\sqrt{r_o^2 + x^2}} \left[\varepsilon^{-\beta t} \sin(\omega t + \varphi - \delta) - \varepsilon^{-\frac{r}{L}t} \sin(\varphi - \delta) \right]$$

FIG. 20.



Heaviside devotes only a few pages out of about one thousand to circuits of concentrated resistance, inductance, capacity and mutual inductance. The rest of the book deals largely with cable problems and more particularly with "ideal cables" where the numerical work becomes fairly simple.

Consider a transmission line as shown in Fig. 20.

Let the constants per unit length of transmission distance be R, L, C, G.

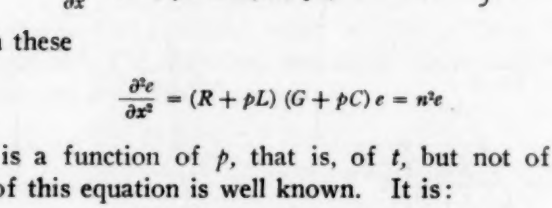
The well-known relations between current and e.m.f. at any point, distance x from the end, is:

$$\frac{\partial i}{\partial x}\,dx = eGdx + Cdx\,\frac{\partial e}{\partial t}$$

and

$$\frac{\partial e}{\partial x}\,dx = iRdx + Ldx\,\frac{\partial i}{\partial t}.$$

FIG. 21.



From these we derive

$$\left.\begin{aligned}\frac{\partial i}{\partial x} &= Ge + C\,\frac{\partial e}{\partial t} = Ge + pCe = e\,(G + pC)\\ \frac{\partial e}{\partial x} &= Ri + pLi = i\,(R + pL)\end{aligned}\right\} \tag{21}$$

and from these

$$\frac{\partial^2 e}{\partial x^2} = (R + pL)\,(G + pC)\,e = n^2 e$$

where n is a function of p, that is, of t, but not of x. The solution of this equation is well known. It is:

$$e = A_1 \varepsilon^{nx} + A_2 \varepsilon^{-nx} \tag{22}$$

$$= A_1\,(\cosh nx + \sinh nx) + A_2\,(\cosh nx - \sinh nx)$$

$$= (A_1 + A_2)\cosh nx + (A_1 - A_2)\sinh nx$$

$$e = K_1 \cosh nx + K_2 \sinh nx \tag{23}$$

where K_1 and K_2 are determined from the terminal conditions.

Since from (21)

$$\frac{di}{dx} = (G + pC)\,e = Ye, i = (G + pC)\int e dx$$

$$\therefore i = \frac{Y}{n}\left[K_1 \sinh nx + K_2 \cosh nx\right] \tag{24}$$

Special Cases.

(1) Open line.

Then $i = 0$ for $x = 0$; $e = E$ for $x = l$.

$$\therefore E = K_1 \cosh nl + K_2 \sinh nl$$

$$0 = \frac{Y}{n}\left[K_1 \sinh 0 + K_2 \cosh 0\right].$$

Since cosh 0 is 1, it is evident that K_2 must be zero.

$$\therefore E = K_1 \cosh nl, \text{ or } K_1 = \frac{E}{\cosh nl}$$

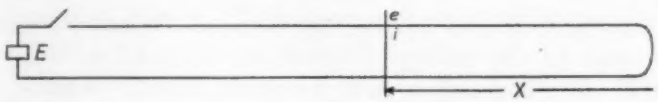
and $$e = E\frac{\cosh nx}{\cosh nl}1$$

and $$i = \frac{Y}{n} \cdot E\frac{\sinh nx}{\cosh nl}1$$

} under Heaviside conditions. (25)

The method of solving equations of this type by the expansion theorem will be shown later.

FIG. 22.



(2) Short-circuited line (Fig. 22).

For $x = 0$, the potential difference = 0, therefore $e = 0$ for $x = 0$, $e = E$ for $x = l$.

From (23),

$$\therefore 0 = K_1 \cosh 0 + K_2 \sinh 0 \therefore K_1 = 0$$

and

$$e = K_2 \sinh nx. \quad \text{For } x = l, e = E$$

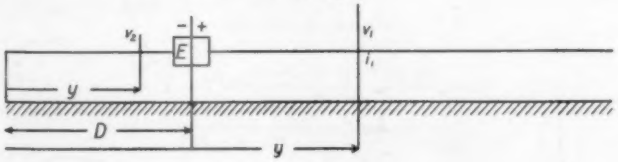
$$\therefore K_2 = \frac{E}{\sinh nl}$$

$$\therefore e = E\frac{\sinh nx}{\sinh nl}1$$

$$i = \frac{Y}{n}E\frac{\cosh nx}{\sinh nl}1$$

} under Heaviside conditions. (26)

FIG. 23.



(3) Consider the potential distribution in a single conductor cable the outer covering of which is grounded.

Let a battery be inserted at distance D from the left terminal, and let in this case the conductor be grounded at the left-hand terminal and insulated at the right.

For the right-hand side:

$$V_1 = K_1 \cosh n(l-y) + K_2 \sinh n(l-y)$$
$$i_1 = \frac{Y}{n}[K_1 \sinh n(l-y) + K_2 \cosh n(l-y)].$$

For $y = l$, $i_1 = 0$, therefore $K_2 = 0$.

$$\therefore V_1 = K_1 \cosh n(l-y).$$

For the left-hand side:

$$V_2 = K_3 \cosh ny + K_4 \sinh ny$$

when $y = 0$, $V_2 = 0$, and therefore $K_3 = 0$.

$$\therefore V_2 = K_4 \sinh ny.$$

At $y = D$ the difference of potential between V_2 and V_1 must be the voltage of the battery. Therefore, $K_1 \cosh n(l-D) - K_4 \sinh nD = E$. Since $\cosh\alpha \cosh\beta + \sinh\alpha \sinh\beta = \cosh(\alpha+\beta)$, it is seen by inspection that this condition obtains if $K_1 = \frac{\cosh nD}{\cosh nl} E$ and $K_4 = -\frac{\sinh n(l-D)}{\cosh nl} E$

Thus

$$\left.\begin{aligned} V_1 &= \frac{\cosh nD}{\cosh nl} \cosh n(l-y)\, E\,1 \\ V_2 &= -\frac{\sinh n(l-D)}{\cosh nl} \sinh ny\, E\,1 \end{aligned}\right\} \quad (27)$$

Heaviside gives this solution in a slightly different form on page 140 (Vol. ii). He uses m instead of n and trigonometric terms instead of hyperbolic.

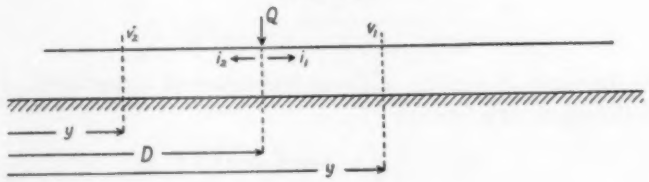
$$\left.\begin{aligned} &\text{Since} && m^2 = -n^2,\ n = jm \\ &\text{and since} && \sinh jm\alpha = j \sin m\alpha \\ &\text{and} && \cosh jm\alpha = \cos m\alpha \end{aligned}\right\} \quad (28)$$

$$\left.\begin{aligned} V_1 &= \frac{\cos mD \cos m(l-y)}{\cos ml} E\,1 \\ V_2 &= \frac{\sin m(l-D)}{\cos ml} \sin my\, E\,1 \end{aligned}\right\} \quad (29)$$

Another interesting application of the operational solution is illustrated in Fig. 24. A charge Q is suddenly applied at a point on an open-circuited cable. In this particular case both ends of the cable are open-circuited and the sheath is grounded. Find the distribution of potential on both sides.

In this case the equations are limited to a conductor—ideal cable—without self-induction, so that there are no wave phenomena. The current diffuses in both directions, i_1 being equal to i_2

FIG. 24.



since the initial current obviously does not know anything about how long each line is. The current is impulsive, that is, $i = pQ\,1$. It is infinite for a very short time, then zero after that time, and is thus of the same shape as $pE1$. Thus Q enters as a unit function.

The solution is obtained as follows: For the right-hand side:

$$V_1 = K_1 \cosh n(l-y) + K_2 \sinh n(l-y)$$

$$i_1 = \frac{Y}{n}[K_1 \sinh n(l-y) + K_2 \cosh n(l-y)].$$

Since for $y = l$, $i_1 = 0$ therefore $K_2 = 0$

$$\therefore V_1 = K_1 \cosh n(l-y)$$

and

$$i_1 = \frac{Y}{n} K_1 \sinh n(l-y).$$

For the left-hand side:

$$V_2 = K_3 \cosh ny + K_4 \sinh ny$$

$$i_2 = \frac{Y}{n}[K_3 \sinh ny + K_4 \cosh ny].$$

For $y = 0$, $i_2 = 0$ and therefore $K_4 = 0$

$$\therefore V_2 = K_3 \cosh ny$$

and

$$i_2 = \frac{Y}{n} K_3 \sinh ny.$$

For $y = D$, $i_1 + i_2 = i = pQ\,1$

$$\therefore \frac{Y}{n}\,[K_1 \sinh n\,(l - D) + K_2 \sinh nD] = p\,Q\,1\,.$$

This is satisfied if

$$K_1 = \frac{\cosh nD}{\sinh nl}\,p\,Q\,\frac{n}{Y}\,1$$

and

$$K_2 = \frac{\cosh n\,(l - D)}{\sinh nl}\,p\,Q\,\frac{n}{Y}\,1$$

$$\left.\begin{aligned} \therefore V_1 &= \frac{\cosh nD}{\sinh nl}\cosh n\,(l - y)\,\frac{npQ}{Y}\,1 \\ V_2 &= \frac{\cosh n\,(l - D)}{\sinh nl}\cosh ny\,\frac{npQ}{Y}\,1 \end{aligned}\right\}. \tag{30}$$

In the case of a cable without leakage and inductance, $Y = pC$ when the equations become

$$\left.\begin{aligned} V_1 &= \frac{\cosh nD}{\sinh nl}\cosh n\,(l - y)\,\frac{pQ}{C}\,1 \\ V_2 &= \frac{\cosh n\,(l - D)}{\sinh nl}\cosh ny\,\frac{pQ}{C}\,1 \end{aligned}\right\} \tag{31}$$

Heaviside gives a large number of similar problems, all of which can readily be solved in the manner indicated above.

Of special interest to engineers are problems involving concentrated as well as distributed inductance, resistance, capacity, and leakage, as examples may be cited such combinations as a cable with a certain terminal impedance, as is shown in Fig. 25, or a transmission line of two insulated conductors. In the first case the capacity and inductance is that between conductor and sheath, in the second case the capacity and inductance between lines.

Referring to equations (23) and (24),

$$e = K_1 \cosh nx + K_2 \sinh nx$$

$$i = \frac{Y}{n}\,[K_1 \sinh nx + K_2 \cosh nx]\,.$$

For $x = 0$, $e = e_1$, and $i = i_1$.

$$\therefore e_1 = K_1 \qquad \text{or} \qquad K_1 = e_1$$

$$i_1 = \frac{Y}{n}\,K_2 \qquad K_2 = \frac{n}{Y}\,i_1$$

$$\therefore e = e_1 \cosh nx + \frac{n}{Y}\,i_1 \sinh nx \tag{32}$$

But $i_1 z_r = e_1$ if z_r is the generalized impedance of the receiving circuit. It is, for instance, $R_0 + pL_0$ in an inductive circuit of resistance R_0 and inductance L_0.

Substituting this value of e_1 in equation (31), we get:

$$e = i_1\left(z_r \cosh nx + \frac{n}{Y} \sinh nx\right)$$

for $x = l$, $e = E_g$, and

$$\therefore i_1 = \frac{E_g}{z_r \cosh nl + \frac{n}{Y} \sinh nl}$$

Therefore,

$$e = E_g \frac{z_r \cosh nx + \frac{n}{Y} \sinh nx}{z_r \cosh nl + \frac{n}{Y} \sinh nl}$$

But $n = \sqrt{(R + pL)(G + pC)}$ and $Y = G + pC$ and $Z = R + pL$.

$$\therefore \frac{Z}{n} = \frac{n}{Y} = z_0 = \sqrt{\frac{R + pL}{G + pC}}$$

and

$$e = E_g \frac{z_r \cosh nx + z_0 \sinh nx}{z_r \cosh nl + z_0 \sinh nl} \tag{33}$$

and by a similar method, we find,

$$i = E_g \frac{\cosh nx + \frac{z_r}{z_0} \sinh nx}{z_r \cosh nl + z_0 \sinh nl} \tag{34}$$

The ratio z' between e and i at any point of the line at any instant is then

$$\frac{e}{i} = z' = \frac{z_r \cosh nx + z_0 \sinh nx}{\cosh nx + \frac{z_r}{z_0} \sinh nx} \tag{35}$$

EXAMPLES OF THE USE OF THE EXPANSION THEOREM.

Consider first the simplest possible case—a plain "ideal cable" short circuited at the receiving end of the line.

From equation (26) or equation (32) (by making $z_r = 0$),

$$e = E \frac{\sinh nx}{\sinh nl} 1,$$

and from equation (28) it is seen that this can also be written

$$e = E \frac{\sin mx}{\sin ml} 1. \tag{36}$$

It is by no means essential to introduce trigonometric functions

instead of hyperbolic, it is done simply because engineers are more familiar with the former.

$$\because Y_{(p)} = \sin mx$$
$$z_{(p)} = \sin ml$$

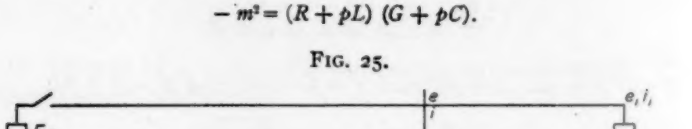
$z_{(p)} = 0$ gives the following roots

$$m_1 = \frac{\pi}{l}, \; m_2 = \frac{2\pi}{l} \cdots\cdots \text{or } m_s = \frac{s\pi}{l}. \tag{37}$$

The root $m = 0$ is taken care of in $\frac{Y_{(0)}}{Z_{(0)}}$, but

$$-m^2 = (R + pL)(G + pC).$$

FIG. 25.



Assume in this case for the sake of brevity that $G = 0$ and $L = 0$. Then

$$-m^2 = pRC. \tag{38}$$

Thus

$$-\frac{s^2\pi^2}{l^2} = pRC \text{ and } p = -\frac{s^2\pi^2}{l^2RC}. \tag{39}$$

Further

$$\frac{dz}{dp} = \frac{dz}{dm}\frac{dm}{dp}$$

and

$$\frac{dz}{dm} = l \cos ml$$

From equation (38) $-2\,mdm = RCdp$, and

$$\therefore \frac{dm}{dp} = -\frac{RC}{2m} = -\frac{RCl}{2s\pi}$$

$$\therefore \frac{dz}{dp} = \frac{RCl^2}{2s\pi} \cos ml$$

and from equation (39)

$$p\frac{dz}{dp} = +\frac{s\pi}{2} \cos ml$$

$$\frac{Y_{(0)}}{z_{(0)}} = \frac{\sin mx}{\sin ml},$$

and as these approach zero $= \frac{mx}{ml} = \frac{x}{l}$. Also

$$Y_{(p)} = \sin \frac{s\pi x}{l}$$

$$\therefore e = E\left[\frac{x}{l} + \Sigma \frac{\varepsilon^{-\frac{s^2\pi^2}{p^2RC}t} \sin \frac{s\pi x}{l}}{\frac{s\pi}{2} \cos \frac{s\pi l}{l}}\right]$$

$$= E\left[\frac{x}{l} + \sum_{s=1,2,3\ldots} (-1)^s \; 2 \frac{\varepsilon^{-\frac{s^2\pi^2}{p^2RC}t} \sin \frac{s\pi}{l}x}{s\pi}\right] \tag{40}$$

Another Example.

Find the voltage at any time and place of the same "ideal" cable if the cable was suddenly connected to an alternator of voltage $e = E \cos \omega t$ instead of to a storage battery.

Note that in this case at $t = 0$, that is when the switch is closed, the maximum value of the impressed voltage is applied to the cable. Then

$$e = E \frac{\sin mx}{\sin ml} \frac{p^2}{\omega^2 + p^2} 1 \tag{41}$$

$z_{(p)} = 0$ gives a set of roots as above,

$$m = \frac{s\pi}{l} \tag{42}$$

and in addition, two roots $\pm j\omega$. It has been shown that the two expressions corresponding to the imaginary roots $\pm j\omega$ give the permanent term.

Write $z_{(p)} = h_{1(p)}h_{2(p)}$ where $h_{1(p)} = \sin ml$ and $h_{2(p)} = p^2 + \omega^2$.

Determine first the permanent term, that is, use the two terms under the summation sign which depend upon $\pm j\omega$.

Referring to the discussion on $h_{1(p)}h_{2(p)}$, we need to determine $ph_{1(p)}h_2'_{(p)}$ corresponding to roots $\pm j\omega$.

$h_{1(p)} = \sin ml$ and $-m^2 = pCR \therefore m = \pm j\sqrt{pCR} = \pm j\sqrt{j\omega RC}$

but

$$\sqrt{j} = \frac{1}{\sqrt{2}}(1 + j) \therefore \pm j\sqrt{j} = \pm \frac{1}{\sqrt{2}}(j - 1)$$

and $$h_1(p) = \sin\left(\pm l(j-1)\sqrt{\frac{\omega RC}{2}}\right) \text{ for } p = j\omega$$

$$h_2'(p) = 2j\omega \text{ for } p = j\omega$$

$$\therefore p h_1(p)\ h_2'(p) = -2\omega^2 \sin\left(\pm l(j-1)\sqrt{\frac{\omega RC}{2}}\right)$$

$$Y_{(p)} = -\omega^2 \sin \pm x(j-1)\sqrt{\frac{\omega RC}{2}} \text{ for } p = j\omega$$

$$\therefore \frac{Y_{(p)}}{p\frac{dz}{dp}} \varepsilon^{pt} = \frac{\sin\left(x(j-1)\sqrt{\frac{\omega RC}{2}}\right)}{2\sin\left(l(j-1)\sqrt{\frac{\omega RC}{2}}\right)} \varepsilon^{j\omega t} \text{ for } p = j\omega \tag{43}$$

Similarly

$$\frac{Y_{(p)}}{p\frac{dz}{dp}} \varepsilon^{pt} = \frac{\sin\left(x(j+1)\sqrt{\frac{\omega RC}{2}}\right)}{2\sin\left(l(j+1)\sqrt{\frac{\omega RC}{2}}\right)} \varepsilon^{-j\omega t} \text{ for } p = -j\omega\,. \tag{44}$$

Since $\sin(\alpha + j\beta) = \sin\alpha\cosh\beta + j\cos\alpha\sinh\beta$ we get after some transformations the permanent term to be:

$$e_{\text{perm}} = E_g \sqrt{\frac{\sin^2 ax\cosh^2 ax + \cos^2 ax\sinh^2 ax}{\sin^2 al\cosh^2 al + \cos^2 al\sinh^2 al}} \cos(\omega t - \psi) \tag{45}$$

where $a = \sqrt{\frac{\omega RC}{2}}$ and

$$\tan\psi = \frac{\cos ax\sin al\cosh al\sinh ax - \sin ax\cos al\sinh al\cosh ax}{\sin ax\sin al\cosh ax\cosh al + \cos ax\cos al\sinh ax\sinh al} \tag{46}$$

To obtain the transient term we proceed to find $ph_{2(p)}h_1'_{(p)}$. The roots now are $m = \frac{s\pi}{l}$ as shown above and $p = -\frac{s^2\pi^2}{l^2RC}$

$$h_2(p) = \omega^2 + p^2 = \omega^2 + \frac{s^4\pi^4}{l^4R^2C^2}\quad ph_1'(p) = \frac{s\pi}{2}\cos ml$$

as has been shown previously.

$$\therefore ph_{2(p)}\ h_1'_{(p)} = \left(\omega^2 + \frac{s^4\pi^4}{l^4R^2C^2}\right)\frac{s\pi}{2}\cos ml$$

$$Y_{(p)} = \frac{s^4\pi^4}{l^4C^2R^2}\sin mx \tag{47}$$

$$e = e_{\text{perm}} + 2\pi^2 \sum_{s=1,2,3\ldots} (-1)^s \frac{s^3 \sin s\pi\frac{x}{l}}{\omega^2 l^2 C^2 R^2 + s^4\pi^4} \varepsilon^{-\frac{s^2\pi^2}{l^2CR}}$$

As another illustration of the use of the expansion theorem, equation (31) will be solved.

$$V_1 = \frac{\cosh nD \cosh n\,(l-y)}{\sinh nl} \frac{nQ}{C} 1$$

Again assume the simplest case, so that we have only resistance and capacity.

Since $\cosh n\alpha = \cosh mj\alpha = \cos m\alpha$ and $\sinh n\alpha = \sinh jm\alpha = j \sin m\alpha$.

$$V_1 = \frac{\cos mD \cos m\,(l-y)}{\sin ml} \frac{mQ}{c} 1 \tag{48}$$

$Z_{(p)} = 0$ gives $m = \frac{s\pi}{l}$, thus since $-m^2 = pCR$, and as has been shown in previous examples,

$$p\frac{dz}{dp} = \frac{s\pi}{2} \cos ml = (-1)^s \frac{s\pi}{2}$$

$$Y_{(p)} = m \cos mD\,(\cos ml \cos my + \sin ml \sin my)$$

but

$$\sin ml = 0$$

$$\cos ml = (-1)^s$$

$$\therefore Y_{(p)} = (-1)^s \cos mD \cos my \frac{s\pi}{l}$$

$$\therefore \frac{Y_{(p)}}{p\frac{dz}{dp}} = \frac{2 \cos mD \cos my}{l}$$

$\frac{Y_{(0)}}{z_{(0)}}$ must then be determined. As p approaches zero m also approaches zero, thus $\sin ml = ml$ and $\cos mD$ or $\cos m\,(l-y)$ approaches unity. Thus

$$\frac{Y_{(0)}}{z_{(0)}} = \frac{mQ}{mCl} = \frac{Q}{CL}$$

$$\therefore V_1 = \frac{Q}{Cl}\left[1 + 2 \sum_{s=1,2,3\ldots} \cos mD \cos my\, \varepsilon^{-\frac{s^2\pi^2}{lCR}t}\right] \tag{49}$$

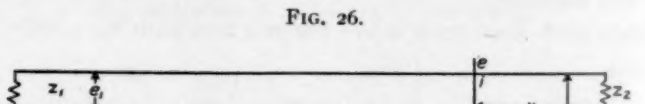
GENERAL TERMINAL CONDITIONS.

It has been shown that when the transmission line is either grounded or open, no difficulty exists in finding the roots $z_{(p)} = 0$. It often becomes quite difficult, however, if the terminal conditions are not as simple.

Heaviside discusses such cases in various parts of his book.

Since, however, his treatment is somewhat different from that given below, it will be well to illustrate one or two cases. Let us take, for instance, the case shown on page 143 (Vol. ii). Referring to Fig. 26, the long horizontal line may be the insulated conductor in a cable whose outside sheath is grounded. z_1 and z_2 are concentrated impedances.

FIG. 26.



From previous consideration it is evident that for the condition in the cable part we have:

$$\left.\begin{aligned} e &= K_1 \cosh nx + K_2 \sinh nx \\ i &= \frac{Y}{n}\left[K_1 \sinh nx + K_2 \cosh nx\right] \end{aligned}\right\} \tag{50}$$

For $x = 0$,

$$e = e_2 = l_2 z_2 = K_1 (1 + 0) = K_1$$

for $x = l$,

$$e = e_1 = K_1 \cosh nl + K_2 \sinh nl$$

But $$E = e_1 + i_1 z_1 \therefore E = i_2 z_2 \cosh nl + K_2 \sinh nl + i_1 z_1 \tag{51}$$

for $x = 0$,

$$i_2 = \frac{Y}{n}\left[0 + K_2\right] \therefore K_2 = \frac{n i_2}{Y} \tag{52}$$

For $x = l$,

$$i_1 = \frac{Y}{n}\left[i_2 z_2 \sinh nl + \frac{n}{Y} i_2 \cosh nl\right] \tag{53}$$

From equations (51) and (53)

$$E = i_2 z_2 \cosh nl + \frac{n}{Y} i_2 \sinh nl + \frac{Y z_1}{n} i_2 \left[z_2 \sinh nl + \frac{n}{Y} \cosh nl\right]$$

$$= i_2 \underbrace{\left[(z_1 + z_2) \cosh nl + \frac{1}{n}\left[z + Y z_1 z_2\right] \sinh nl\right]}_{A} = i_2 A \tag{54}$$

$$\therefore i_2 = \frac{E}{A}$$

Therefore from equation (50)

$$e = \frac{z_2 \cosh nx + \frac{n}{Y} \sinh nx}{(z_1 + z_2) \cosh nl + \frac{1}{n}(z + Y z_1 z_2) \sinh nl} E1 \tag{55}$$

From equation (50), since $\frac{z}{n} = \frac{n}{Y} = z_0$

$$i = \frac{Y}{n} i_2 \left[Z_2 \sinh nx + \frac{n}{Y} \cosh nx \right]$$

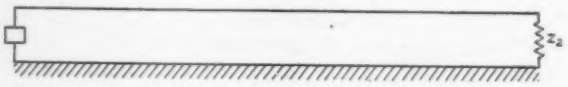
$$= \frac{\frac{z_2}{z_0} \sinh nx + \cosh nx}{(z_1 + z_2) \cosh nl + \frac{1}{n} [z + Y z_1 z_2] \sinh nl} E\,1 \qquad (56)$$

Special case $z_1 = 0$.

$$\therefore e = \frac{z_2 \cosh nx + \frac{n}{Y} \sinh nx}{z_2 \cosh nl + \frac{z}{n} \sinh nl} E\,1$$

$$= \frac{z_2 \cosh nx + z_0 \sinh nx}{z_2 \cosh nl + z_0 \sinh nl} E\,1 \qquad (57)$$

FIG. 27.



Where

$$z_0 = \frac{n}{Y} = \frac{z}{n}$$

From (56) for $z_1 = 0$.

$$i = \frac{\frac{z_2}{z_0} \sinh nx + \cosh nx}{z_2 \cosh nl + z_0 \sinh nl} E\,1 \qquad (58)$$

At the receiving end $x = 0$.

$$\therefore e_2 = \frac{1}{\cosh nl + \frac{z_0}{z_2} \sinh nl} E\,1 \qquad (59)$$

and

$$i_2 = \frac{1}{z_2 \cosh nl + z_0 \sinh nl} E\,1 \qquad (60)$$

The current will be calculated in order to show the procedure when the roots $z_{(p)} = 0$ cannot readily be attained except by graphical means.

Assume again, for the sake of simplicity, that we deal with the "ideal" cable.

$$\therefore n^2 = pCR$$

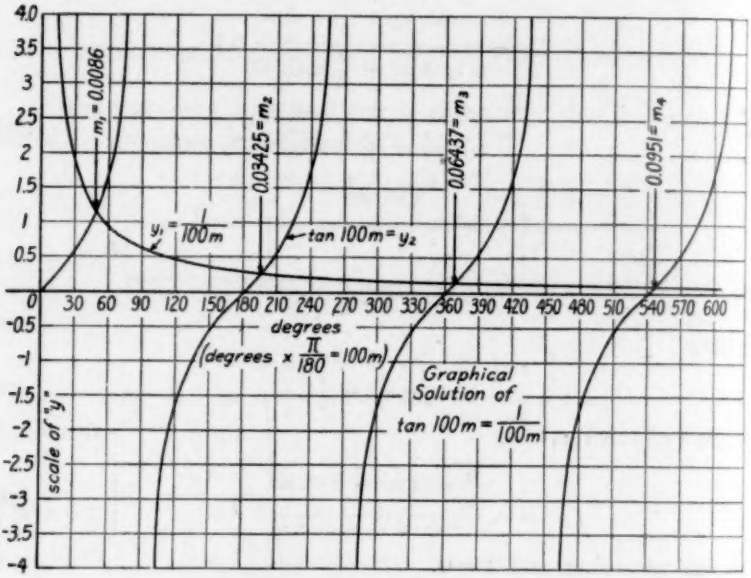
If $m^2 = -n^2$ then $m^2 = -pCR$ and $p = -\frac{m^2}{CR}$.

Let the terminal impedance be a condenser of capacity c_1.

$$\therefore z_1 = \frac{1}{pc_1} = -\frac{CR}{c_1 m^2} \text{ and } z_0 = \frac{z}{n} = \frac{R}{\sqrt{pCR}} = \sqrt{\frac{R}{C}}\frac{\sqrt{CR}}{jm} = \frac{R}{jm}$$

Thus, since cosh $jml = \cos ml$ and sinh $jml = j \sin ml$

FIG. 27a.



$$i_2 = \frac{1}{-\frac{cR}{c_1 m^2}\cos ml + \frac{R}{m}\sin ml} E1$$

$$= \frac{\frac{c_1}{CR}m^2}{-\cos ml + \frac{c_1}{C} m \sin ml} E1 \qquad (61)$$

$z_{(p)} = 0$ gives $\tan ml = \frac{C}{c_1 m}$.

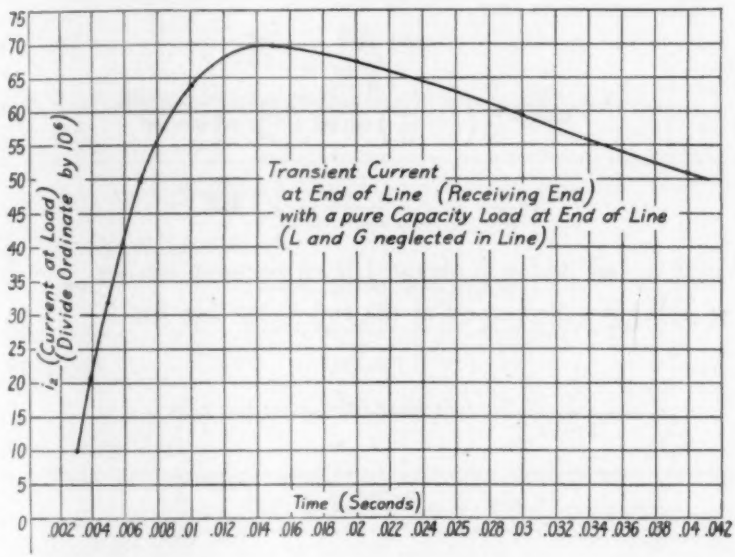
What is wanted then are all possible values of m, m_1, m_2, m_3, etc., which satisfy the relation $\tan ml = \frac{C}{c_1 m}$.

Fig. 27 illustrates the method of obtaining these roots in a

particular numerical case. An "ideal" (100 miles long) cable of resistance 88 ohms per mile of capacity .054 mf. per mile is shunted at the distant end by a condenser c_1 of the same capacity as that of the entire cable. Find the equation for the current at the distant end when the cable is suddenly connected to a source of constant potential E.

The procedure used in finding the roots is to assign arbitrarily values to m, then plot the corresponding tangent curves and the graph $\frac{C}{c_1 m}$.

FIG. 27b.



Where these curves intersect the roots are obviously located. These roots appear to be $m_1 = .0086$, $m_2 = .0343$, $m_3 = .0643$, etc. To find i_2, the current at the receiving circuit, we proceed in the usual way.

In this case

$$n^2 = (R + pL)(G + pC) = pCR \therefore -m^2 = pCR \qquad (62)$$

$$p\frac{dz}{dp} = p\frac{dz}{dm}\frac{dm}{dp}$$

$$\frac{dz}{dm} = l \sin ml + \frac{c_1}{C}\left[ml \cos ml + \sin ml\right]$$

from equation (62)

$$\frac{dm}{dp} = -\frac{CR}{2m}$$

$$\therefore p\frac{dz}{dp} = \frac{m}{2}\left(l + \frac{c_1}{C}\right)\sin ml + \frac{c_1}{2C}m^2 l\cos ml$$

$\frac{Y_{(o)}}{z_{(o)}}$ corresponds to the steady case. Since we have a condenser at the end of the line, the final direct current is of course zero, thus $\frac{Y_{(o)}}{z_{(o)}} = 0$. Or, it can be seen directly by making $p = 0$, which means $m = 0$.

The solution is thus,

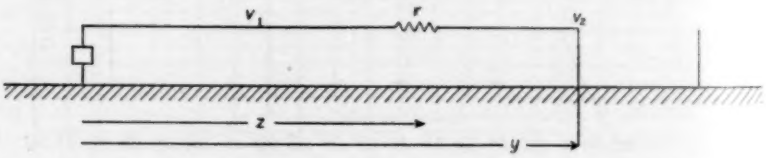
$$i = \sum_{m_1 m_2 m_3} \frac{\frac{c_1}{CR}m^2\,\varepsilon^{-\frac{m^2}{CR}t}}{\frac{m}{2}\left(l + \frac{c_1}{C}\right)\sin ml + \frac{c_1}{2C}m^2 l\cos ml}E$$

$$= \frac{2E}{RL}\sum_{\alpha}\frac{\alpha\cos\alpha}{\alpha + \sin\alpha\cos\alpha}\varepsilon^{-\frac{\alpha^2}{RCL^2}t}$$

Where $\alpha = ml$ and α is obtained from equation $\tan\alpha = \frac{C}{c_1 m} = \frac{Cl}{c_1\alpha}$

Fig. 27b gives the current at the end of the line for $E = 1$.

FIG. 28.



Two different cables connected together by a resistance, Fig. 28.

For the right-hand cable:

$$V_2 = K_1\cosh n_2(l-y) + K_2\sinh n_2(l-y)$$

For $y = l$, $\quad V_2 = 0 \therefore K_1 = 0$

$$\therefore V_2 = K_2\sinh n_2(l-y)$$

$$i_2 = \frac{Y_2}{n_2}K_2\cosh n_2(l-y)$$

$$\therefore \frac{V_2}{i_2} = \frac{n_2}{Y_2}\tanh n_2(l-y)$$

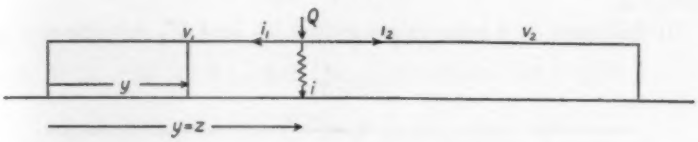
for $y = z$, $\quad \frac{V_2}{i_2} = \frac{n_2}{Y_2}\tanh n_2(l-z) = z$

This z may then be regarded as a resistance operator. Thus the total resistance operator at the end of the first cable is $z_1 = r + \frac{n_1}{Y_1}$ tanh $n_2(l-z)$. Thus the potential and current in the first cable can be found from equations (33) and (34) when we substitute for $z_1 = r + \frac{n_2}{Y_2}$ tanh $n_2\ (l-z)$, and for l the length of the left-hand cable.

The following problem will show how a branch circuit is treated.

An "ideal" cable is grounded at both ends and a charge Q is suddenly applied at $y=z$, where also there is a "leak" to ground.

FIG. 29.



For the left-hand side:

$$V_1 = K_1 \cosh ny + K_2 \sinh ny$$

for $y = 0$, $V_1 = 0$ thus $K_1 = 0$ and $V_1 = K_2 \sinh ny$.

Similarly,

$$V_2 = K_2' \sinh n\,(l-y)$$

thus i_1 at $y=z$ is

$$i_1 = \frac{Y}{n} K_2 \cosh nz$$

and i_2 at $y=z$ is

$$i_2 = \frac{Y}{n} K_2' \cosh n\,(l-z)$$

At $y=z$,

$$V_1 = V_2 \text{ thus } K_2 \sinh nz = K_2' \sinh n\,(l-z)$$

$$\therefore K_2' = K_2 \frac{\sinh nz}{\sinh n\,(l-z)}$$

$$\therefore i_2 \text{ at } z = \frac{Y}{n} K_2 \frac{\sinh nz}{\sinh n\,(l-z)} \cosh n\,(l-z)\,.$$

The "leak" current is $i = \frac{V_2}{r} = K_2 \frac{\sinh nz}{r}$.

Since the supplied current is pQ and $pQ = i_1 + i_2 + i$, we get

$$pQ = K_2\left[\frac{Y}{n}\left(\cosh nz + \frac{\sinh nz}{\sinh n(l-z)}\cosh n(l-z)\right) + \frac{\sinh nz}{r}\right]$$

$$= K_2\left[\frac{Y}{n}\frac{\sinh nl}{\sinh n(l-z)} + \frac{\sinh nz}{r}\right]$$

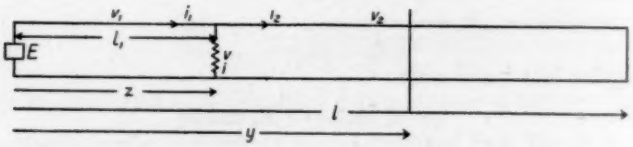
$$\therefore K_2 = \frac{pQ}{\frac{Y}{n}\frac{\sinh nl}{\sinh n(l-z)} + \frac{\sinh nz}{r}} = \frac{-\frac{m^2}{cR}}{\frac{jm}{R}\frac{\sin ml}{\sin m(l-z)} + \frac{\sin mz}{r}}$$

$$\therefore V_1 = \frac{-\frac{m^2}{cR}\sin my}{\frac{m}{R}\frac{\sin ml}{\sin m(l-z)} + \frac{\sin mz}{r}}Q\,1\cdot$$

Another problem involving leak at distance z from the generator.

In this case of a transmission line V_1 and V_2 are the potential

FIG. 30.



differences between the two lines. Since the end of the line is short circuited, $V_2 = 0$ for $y = l$,

$$V_2 = K_1 \cosh n_2(l-y) + K_2 \sinh n_2(l-y)$$

For $l - y$, $V_2 = 0$ thus $K_1 = 0$

and $V_2 = K_2 \sinh n(l-y)$

At $y = z$, $V_2 = K_2 \sinh n_2(l-z)$ (a)

and $i_2 = \frac{Y_2}{n_2} K_2 \cosh n_2(l-z)$

$$i = \frac{V_2}{r} = K_2\frac{\sinh n_2}{r}(l-z)$$

$$\therefore i_1 \text{ at } z = i + i_2 = K_2\left[\frac{Y_2}{n_2}\cosh n_2(l-z) + \frac{\sinh n_2(l-z)}{r}\right]$$

the resistance operator which replaces the right-hand cable and the leak is thus

$$\frac{V_2}{i} = \frac{\sinh n_2(l-z)}{\frac{Y_2}{n_2}\cosh n_2(l-z) + \frac{\sinh n_2(l-z)}{r}} = z_r$$

The current and voltage in any part of the left-hand cable can

thus be obtained if this value of z_r is substituted in equations (33) and (34) if at the same time the length of the left-hand cable z_1 is substituted for l in equations (33) and (34).

The current and voltage in the right-hand side can be determined after K_2 is known. K_2 is determined from the modified equation (33) and equation (a) above. When in the modified equation (33) $x = 0$ and $l = z$ and in (a) $y = z$.

CABLE PROBLEM INVOLVING BESSEL'S FUNCTION.

Let the resistance and inductance per unit length vary inversely as the distance x and let the capacity and leakage vary as the distance x. Thus

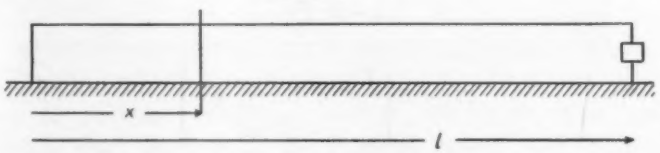
$$R + pL = \frac{R_0 + pL_0}{x}$$

and

$$G + pc = (G_0 + pC_0)\, x$$

where R_0, L_0, G_0 and C_0 are the constants at the origin $x = 0$.

FIG. 31.



$$\frac{\partial e}{\partial x} = i\,\frac{R_0 + pL_0}{x} \quad \therefore i = \frac{x}{R_0 + pL_0}\,\frac{\partial e}{\partial x} \qquad (a)$$

$$\frac{\partial i}{\partial x} = ex\,(G_0 + pC_0) \qquad (b)$$

Differentiate i in respect to x,

$$\therefore \frac{\partial i}{\partial x} = \frac{1}{R_0 + pL_0}\left[x\frac{\partial^2 e}{\partial x^2} + \frac{\partial e}{\partial x}\right]$$

Substitute this in (b), thus

$$x\frac{\partial^2 e}{\partial x^2} + \frac{\partial e}{\partial x} = (R_0 + pL_0)\,(G_0 + pC_0)\,ex = -\alpha_0^2 ex$$

or

$$\frac{d^2 e}{dx^2} + \frac{1}{x}\frac{de}{dx} + \alpha_0^2 e = 0 \qquad (c)$$

Equation (c) is the typical form of the second order Bessel's differential equation. The solution of which is

$$e = AJ_0(\alpha_0 x) + BY_0(\alpha_0 x)$$

where $J_o(\alpha_o x)$ is a zero order *Bessel's function* of x of the first kind and $Y_o(\alpha_o x)$ is a zero order *Bessel's function* of the second kind and A and B are arbitrary constants. In the case of ideal cable

$$n^2 = pCR = pC_o x \frac{R}{x} = pC_o R_o$$

$$\therefore \alpha_o^2 = -pCR_o$$

In that case since with our usual notations $-m^2 - pCR$, we have $\alpha_o^2 - m^2$ or $\alpha_o - m$.

$$\therefore e = AJ_o(mx) + BY_o(mx)$$

For $x - o$, $Y_o(mx) - -\infty$, therefore B must be zero. Thus see table of Jahnke and Emde, page 124.

$$\therefore e = AJ_o(mx)$$

Since the battery is located at $x - l$ where $e - E$,

$$A = \frac{E}{J_o(ml)}$$

$$\therefore e = E\frac{J_o(mx)}{J_o(ml)}$$

$$\therefore e = E\left[\frac{Y_{(o)}}{z_o} + \Sigma \frac{Y_{(p)}}{p\frac{dz}{dp}} \varepsilon^{pt}\right]$$

Since $R = \frac{R_o}{x}$ it means that for $x - o$, $R - \infty$, thus the left-hand side of the line is open circuited. Thus the permanent voltage is E. Thus

$$e = E\left[1 + \Sigma \frac{Y_{(p)}\,\varepsilon^{pt}}{p\frac{dz}{dp}}\right]$$

Since $$-m^2 = pC_o R_o$$

$$p\frac{d}{dp}J_o(ml) = p\frac{d}{dm}J_o(ml) \times \frac{dm}{dp} = -\frac{m}{2}$$

$$\therefore e = E\left[1 + \Sigma \frac{Y_{(p)}\,\varepsilon^{pt}}{\frac{m}{2} l J_1(ml)}\right]$$

where $$J_1(x) = -\frac{d}{dx}J_o(x)$$

Solutions involving Bessel's functions appear often in connection with current distribution in cylindrical conductors.

Such a problem might be, to determine the flux and current distribution in a conductor surrounded by a coil to which suddenly an e.m.f. is applied.

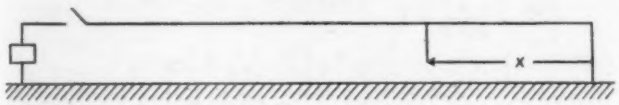
In this case the solution is simplified if we assume that the time constant of the coil is negligible, so that the current suddenly rises to its maximum value and then remains constant.

DISCONNECTING A CABLE.

Before the switch is opened the current is in this example assumed to have reached its steady state, that is, $I = \frac{E}{Rl}$ in all parts of the cable. After the switch is opened, the current at the switch is zero.

If, therefore, we determine the expression of some peculiar e.m.f. impressed at the generator such that at the generator the

FIG. 32.



current is constant, equaling $-I$ at all times after closing the switch, then the sum of the original current and the current caused by this peculiar e.m.f. will obviously be the current in any part of the cable.

It has been shown that with the receiving end of a cable grounded

$$e = e_g \frac{\sinh nx}{\sinh nl}$$

and

$$i = \frac{Y}{n} e_g \frac{\cosh nx}{\sinh nl}$$

$$\therefore i_g = e_g \frac{Y}{n} \frac{\cosh nl}{\sinh nl}$$

The peculiar e.m.f. which makes $i_g = -I1$ is then

$$e_g = -\frac{n}{Y} \frac{\sinh nl}{\cosh nl} I1$$

and thus the current in any part of the cable due to this e.m.f. is:

$$i_1 = -\frac{Y}{n} \frac{\cosh nx}{\sinh nl} \frac{n}{Y} \frac{\sinh nl}{\cosh nl} I1 = -\frac{\cosh nx}{\cosh nl} I1$$

The total current is then

$$i = i_1 + I = I\left[1 - \frac{\cosh nx}{\cosh nl}1\right]$$
$$= \frac{E}{Rl}\left[1 - \frac{\cosh nx}{\cosh nl}1\right]$$

At the end of the line $x = o$

$$\therefore i_r = \frac{E}{Rl}\left[1 - \frac{1}{\cosh nl}1\right]$$

Solve
$$i_1 = -\frac{I}{\cosh nl}1 = -\frac{Y_{(p)}I}{z_{(p)}}1$$

$$Y_{(p)} = 1$$

$$z_{(p)} = \cosh nl = \cos ml$$

where
$$-m^2 = n^2 = pCR$$

$$p\frac{dz}{dp} = p\frac{dz}{dm}\frac{dm}{dp} = -\frac{ml}{2}\sin ml$$

$$\frac{Y_{(o)}}{z_{(o)}} = \frac{1}{\cos o} = 1$$

$$\therefore i_1 = -\left[I + \Sigma\frac{2I\varepsilon^{pt}}{-ml\sin ml}\right]$$

$z_{(p)} = o$ gives $\cos ml = o$ $\therefore ml = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots \frac{k\pi}{2}$

or

$$ml = \frac{2s+1}{2}\pi$$

$$\therefore p = -\left(\frac{2s+1}{2}\right)^2\frac{\pi^2}{CRl^2}$$

$$\therefore i_1 = -\left[I - \sum_{s=o}^{s=\infty}\frac{2I\varepsilon^{pt}}{\frac{2s+1}{2}\pi\sin\frac{2s+1}{2}\pi}\right]$$

$$\therefore i = I + i_1 = \frac{4E}{RL\pi}\sum_{s=o}^{s=\infty}\varepsilon^{-\alpha t}\frac{\cos s\pi}{2s+1}$$

where
$$\alpha = \left(\frac{2s+1}{2}\right)^2\frac{\pi^2}{CRl^2}$$

THE DISCHARGE OF A CHARGED CABLE.

Switch a is closed from $t = o$ to $t = t_1$, at which time switch b is closed. Find the current in the cable. Short circuiting the battery means that the impressed e.m.f. is zero at time t_1. This again is equivalent to leaving the original battery on, but applying a

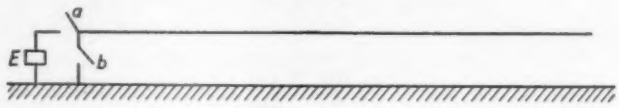
battery of potential $-E1$ at time t. The total current is then the sum of the currents due to these two e.m.f.'s.

Current due to $E1 - i_1 = \frac{Y}{n} E \frac{\sinh nx}{\cosh nl} 1 = E \frac{Y_{(p)}}{z_{(p)}} 1$, as is shown in equation (25).

This is readily solved by the expansion theorem, which gives

$$i_1 = \frac{2E}{Rl} \sum_0^\infty (-1)^s \sin\left(\frac{2s+1}{2} \pi \frac{x}{l}\right) \varepsilon^{\beta t}$$

FIG. 33.



When e.m.f. $-E1$ is applied at $t = t_1$, the current is obviously

$$i_2 = -\frac{2E}{Rl} \sum_0^\infty (-1)^s \sin\left(\frac{2s+1}{2} \pi \frac{x}{l}\right) \varepsilon^{\beta(t-t_1)}$$

$$\therefore i = \frac{2E}{Rl}\left[\sum_0^\infty (-1)^s \sin\left(\frac{2s+1}{2} \pi \frac{x}{l}\right) \varepsilon^{\beta t} - \sum_0^\infty (-1)^s \sin\left(\frac{2s+1}{2} \pi \frac{x}{l}\right) \varepsilon^{\beta(t-t_1)}\right]$$

where

$$\beta = \left(\frac{2s+1}{2}\right)^2 \frac{\pi^2}{RCl^2}.$$

The corresponding solution, if the cable were connected to earth at the receiving end, is easily found to be

$$i = \frac{2E}{RL}\left[\sum_1^\infty (-1)^s \cos\left(s\pi \frac{x}{l}\right) \varepsilon^{\alpha t} - \sum_1^\infty (-1)^s \cos\left(s\pi \frac{x}{l}\right) \varepsilon^{\alpha(t-t_1)}\right]$$

where

$$\alpha = \frac{s^2\pi^2}{CRl^2}.$$

FRACTIONAL DERIVATIVES.

The second volume of Heaviside's e.m.t. contains a great deal of suggestive matter in reference to the interpretation of divergent series and fractional derivatives. It is quite evident, however, that more work will have to be done on divergent series before it can be of direct use to engineers.

That fractional derivatives have physical significance is, however, evident and in some cases, as, for instance, when an infinitely long cable is considered, it is easily interpreted.

Referring to Fig. 34,

$$\frac{\partial e}{\partial x} = -i(R + pL)$$

$$\frac{\partial i}{\partial x} = -e(G + pC)$$

Thus

$$\frac{\partial^2 e}{\partial x^2} = YZe = n^2 e$$

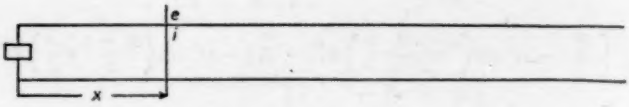
$$\therefore e = A\varepsilon^{nx} + B\varepsilon^{-nx}$$

for $x = \infty$, $e = 0$, therefore $A = 0$ and $e = B\varepsilon^{-nx}$; for $x = 0$, $e = E$, therefore $B = E$ and

$$e = E\varepsilon^{-nx}\,1$$

$$i = \frac{Y}{n} E\varepsilon^{-nx}\,1$$

FIG. 34.



Therefore $\frac{e}{i}$ = resistance operator on unit function is $= \frac{n}{Y}\varepsilon^{-nx}$ at any point of the line.

In the case of an "ideal" cable, $Y = pC$, $Z = R$ and

$$\frac{n}{Y} = \sqrt{\frac{pCR}{pC}} = \left(\frac{R}{pC}\right)^{\frac{1}{2}}$$

The current at the battery is thus

$$i_0 = \left(\frac{pC}{R}\right)^{\frac{1}{2}} E\,1$$

and

$$\frac{i}{i_0} = \frac{\frac{Y}{n} E\varepsilon^{-nx}}{\frac{Y}{n} E} = \varepsilon^{-nx}$$

thus

$$i = E\varepsilon^{-nx}\left(\frac{pC}{R}\right)^{\frac{1}{2}} 1.$$

A very similar problem in heat conduction through an infinitely long bar had many years previously been worked out with con-

ventional mathematics involving Fourier's series. The differential equation relating to the temperature is:

$$\frac{d\theta}{dt} = h^2 \frac{d^2\theta}{dx^2} \qquad (a)$$

where h^2 is termed by Kelvin the diffusivity and by Maxwell the thermometric conductivity.

The rate of heat transferred through unit area is

$$W = -K \frac{d\theta}{dx} \qquad (b)$$

where K is the thermal conductivity of the substance.

When these equations are solved subject to the condition that at the origin $x = 0$, the temperature is kept constant at θ_0 degrees, the rate of flow of heat at the origin becomes

$$W_0 = \frac{K\theta_0}{h\sqrt{\pi t}} \qquad (c)$$

Heaviside compared this problem with the problem given above in the case of the infinite ideal cable, when

$$\frac{\partial e}{\partial x} = -iR \qquad (d)$$

and

$$\frac{\partial i}{\partial x} = -Cpe = -C\frac{de}{dt}$$

which two equations combined give:

$$\frac{\partial^2 e}{dx^2} = RC\frac{de}{dt} \text{ or } \frac{de}{dt} = \frac{1}{RC}\frac{d^2e}{dx^2}. \qquad (e)$$

Comparing (a) and (e), we note that $h^2 = \frac{1}{RC}$. Comparing (b) and (d), we note that $K = \frac{1}{R}$ and $W = i$ and $\theta_0 = E1$ and

$$\frac{K}{h} = \sqrt{\frac{C}{R}} = \left(\frac{C}{R}\right)^{\frac{1}{2}}.$$

The operational solution of the current given above was

$$i = \left(\frac{pC}{R}\right)^{\frac{1}{2}} E1 = \left(\frac{C}{R}\right)^{\frac{1}{2}} Ep^{\frac{1}{2}}1$$

or at the origin

$$i = \left(\frac{C}{R}\right)^{\frac{1}{2}} Ep^{\frac{1}{2}}1 \text{ for } x = 0$$

or substituting heat constants

$$W_0 = \frac{K}{h} Q_0 p^{\frac{1}{2}}1$$

but it is shown in (c) to be

$$W_o = \frac{K}{h}\frac{Q_o}{\sqrt{\pi t}}.$$

Thus Heaviside concludes that

$$p^{\frac{1}{2}}1 = \frac{1}{\sqrt{\pi t}} = (\pi t)^{-\frac{1}{2}}.$$

Therefore, the current at the battery is:

$$i_o = \sqrt{\frac{C}{R\pi t}}E.$$

Incidentally

$$p^{\frac{3}{2}}1 = pp^{\frac{1}{2}}1 = p(\pi t)^{-\frac{1}{2}} = -\frac{1}{2}\pi^{-\frac{1}{2}}t^{-\frac{3}{2}} = -\frac{t^{-\frac{3}{2}}}{2\pi^{\frac{1}{2}}}$$

$$p^{\frac{3}{2}}1 = \frac{1.3t^{-\frac{3}{2}}}{2.2\pi^{\frac{1}{2}}}$$

Similarly

$$p^{-\frac{1}{2}}1 = p^{-1}p^{\frac{1}{2}}1 = \int_0^t (\pi t)\,dt^{-\frac{1}{2}} = \frac{2t^{\frac{1}{2}}}{\pi^{\frac{1}{2}}}$$

$$p^{-\frac{5}{2}}1 = \frac{2^2t^{\frac{5}{2}}}{1.3.5\pi^{\frac{1}{2}}}$$

Fractional derivatives appear apparently only in some problems involving infinitely long conductors. Returning now to the original problem of finding the current at any point and time in an infinitely long " ideal " cable, we have

$$\frac{i}{i_o} = \frac{\frac{Y}{n}E\varepsilon^{-nx}}{\frac{Y}{n}E} = \varepsilon^{-nx}$$

$$\therefore i = E\sqrt{\frac{C}{R}}\varepsilon^{-nx}p^{\frac{1}{2}}1 = E\sqrt{\frac{\varepsilon}{\pi Rt}}\varepsilon^{-\sqrt{\frac{CR}{\pi t}}}$$

ON THE USE OF DIVERGENT SERIES.

In many problems it is of great advantage to expand a function in ascending and descending powers of the independent variable t, and to use one or the other expansion depending upon which gives the least numerical work for a given accuracy.

It should be remembered, however, that if with rising powers of t the series is converging, we can, by taking sufficient number of terms, get any degree of accuracy, whereas with the divergent series, there is a definite limit to the attainable accuracy in every particular problem. This is illustrated by Heaviside in the case of an infinitely long cable which has inductance as well as capacity and resistance.

The relation $i = \frac{Y}{n} E\varepsilon^{-nx} 1$ is obtained as in the previous problem.

Consider, for the sake of simplicity, that the current at the battery is desired, then $x = 0$ and

$$i_0 = \frac{Y}{n} E 1 = \frac{pC}{\sqrt{pC(R+pL)}} E1 = E\left(\frac{pC}{R+pL}\right)^{\frac{1}{2}} 1$$

$$= \frac{E}{\left(\frac{R}{pC}+\frac{L}{C}\right)^{\frac{1}{2}}} 1 = \sqrt{C}\,\frac{E}{(Rp^{-1}+L)^{\frac{1}{2}}} 1$$

If $\alpha = \frac{R}{2L}$, then

$$i_0 = \frac{\sqrt{C}}{\sqrt{L}}\,\frac{E}{(2\alpha p^{-1}+1)^{\frac{1}{2}}} 1 = \frac{E}{Lv(1+2\alpha p^{-1})^{\frac{1}{2}}} 1 \qquad (a)$$

Where

$$v = \frac{1}{\sqrt{LC}}$$

Now expand equation (a) by the binomial series in rising powers of $\frac{\alpha}{p}$. Since

$$\frac{1}{\sqrt{1+x}} = 1 - \frac{x}{2} + \frac{1\cdot 3}{2\cdot 4}x^2 - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}x^3 + \ldots$$

$$i = \frac{E}{Lv}\left[1 - \frac{\alpha}{p} + \frac{1\cdot 3}{\lfloor 2}\left(\frac{\alpha}{p}\right)^2 - \frac{1\cdot 3\cdot 5}{\lfloor 3}\left(\frac{\alpha}{p}\right)^3 + \ldots\right] 1$$

Substituting the values for $\frac{1}{p}1$, $\frac{1}{p^2}1$, etc., we get:

$$i = \frac{E}{Lv}\left[1 - \alpha l + \frac{1\cdot 3}{(\lfloor 2)^2}(\alpha l)^2 - \frac{1\cdot 3\cdot 5}{(\lfloor 3)^2}(\alpha l)^3 + \ldots\right] \qquad (b)$$

This is a convergent solution in rising powers of t. Heaviside then suggests a similar treatment, but with descending powers of t.

If the numerator, as well as the denominator, is multiplied by $\left(\frac{p}{2\alpha}\right)^{\frac{1}{2}}$, we get from equation (a),

$$i_o = \frac{E\left(\frac{p}{2\alpha}\right)^{\frac{1}{2}}}{Lv\left(1+\frac{p}{2\alpha}\right)^{\frac{1}{2}}}\,1\cdot$$

Now expand the denominator by the binomial theorem, thus

$$i_o = \frac{E}{Lv}\left[1-\frac{p}{4\alpha}+\frac{1\cdot 3}{2\cdot 4}\left(\frac{p}{2\alpha}\right)^2-\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\left(\frac{p}{2\alpha}\right)^3+\cdots\right]\left(\frac{p}{2\alpha}\right)^{\frac{1}{2}}1$$

Here is a case of plain differentiations of $\left(\frac{p}{2\alpha}\right)^{\frac{1}{2}}1\cdot$ But

$$\left(\frac{p}{2\alpha}\right)^{\frac{1}{2}}1 = \frac{1}{\sqrt{2\pi\alpha t}}$$

thus the second term in the series becomes

$$-\frac{1}{4\alpha}\frac{d}{dt}\frac{1}{\sqrt{2\pi\alpha t}} = \frac{t^{-\frac{3}{2}}}{8\alpha\sqrt{2\pi\alpha}} = \frac{1}{8\alpha t\sqrt{2\alpha\pi t}}.$$

By a similar differentiation of the other terms, we get:

$$i_o = \frac{E}{Lv}\frac{1}{\sqrt{2\pi\alpha t}}\left[1+\frac{1^2}{8\alpha t}+\frac{1^2 3^2}{\lfloor 2\,(8\alpha t)^2}+\frac{1^2 3^2 5^2}{\lfloor 3\,(8\alpha t)^3}\cdots\right] \qquad (c)$$

This is a divergent series and is easier to evaluate than (b) if αt is large.

It will be found that when substituting such large values the series converges for a while as we add terms, then begins to diverge and increases indefinitely.

Heaviside says (page 237, Vol. ii): "Seek the place where the smallest term occurs, then the error is limited to the size of the smallest term."

To illustrate this numerically, assume that $8\alpha t = 8$, then

$$i_o = K[1+.125+.0351+.0735+\cdots]$$

The sum then of the first three terms = 1.1601 with an error less than .0351.

The proof of this simple rule, as far as I can see, is not given directly by Heaviside, but may be found in "Modern Analysis" by Whittaker and Watson.

GENERALIZED DIFFERENTIATION AND INTEGRATION.

The problem cited above involving infinitely long "ideal" cable probably was instrumental in Heaviside's interest in fractional derivatives in general, and he proves the validity from several points of view (see, for instance, p. 288, Vol. ii). Of particular interest is his generalized differentiation.

Consider first ordinary derivatives of any power of t,

$$\frac{d}{dt}\frac{t^n}{\lfloor n} = \frac{t^{n-1}}{\lfloor n-1} \text{ or } \frac{d}{dt}t^n = \lfloor n \frac{t^{n-1}}{\lfloor n-1}$$

or more general

$$\frac{d^m}{dt^m}\frac{t^m}{\lfloor n} = \frac{t^{n-m}}{\lfloor n-m}$$

or

$$p^m \frac{t^m}{\lfloor n} = \frac{t^{n-m}}{\lfloor n-m} \qquad (a)$$

As long as m is a positive integer and $n \geqq m$, it is readily possible to determine the value; when $m > n$, all derivatives become zero because factorial of negative integers are all minus infinity.

Consider next $p^{-m}t^n$, p^{-m} meaning integration m times. As long as we limit ourselves to functions the integral of which is zero for $t = 0$, in other words, as long as we limit ourselves to functions that are zero at $t = 0$, it is evident that $\frac{1}{p}\,p = 1$ or $p^{-1}p^1 = 1$, thus in such problems p^{-m} is the inverse process of p^m.

$$\therefore p^{-m}\frac{t^n}{\lfloor n} = \frac{t^{n+m}}{\lfloor n+m}. \qquad (b)$$

To illustrate the use of equations (a) and (b), let $m = 2$ and $n = 3$.

$$\therefore p^2 t^3 = \frac{d^2}{dt^2}t^3 = \lfloor 3 \frac{t^1}{\lfloor 1} = 6t$$

$$p^{-2}t^3 = \int_0^t\int_0^t t^3 dtdt = \lfloor 3 \frac{t^5}{\lfloor 5}$$

m will now not be restricted but permitted to be positive or negative, integer or a fraction; then we have

$$\frac{d^m}{dx^m}\frac{x^n}{\lfloor n} = \frac{x^{n-m}}{\lfloor n-m}$$

$\lfloor n-m$ is in general not a whole number but a fraction and has then to be evaluated from the Π function.

$$\Pi(x) = \overline{|(x+1)} = \lim_{n=\infty} \frac{1 \cdot 2 \cdot 3 \ldots (n-1)\, n}{(x+1)(x+2)\ldots(x+n)}\, n^x$$

$$\Pi(o) = 1; \Pi(n) = \overline{|n}; \frac{1}{\Pi(-n)} = 0; \Pi(x) = x\Pi(x-1)$$

$$\Pi(-x)\,\Pi(x-1) = \frac{\pi}{\sin \pi x}$$

$$\Pi\left(-\tfrac{1}{2}\right) = \sqrt{\pi}; \Pi\left(\tfrac{1}{2}\right) = \frac{\sqrt{\pi}}{2}; \Pi\left(-\tfrac{3}{2}\right) = -2\sqrt{\pi}; \Pi\left(\tfrac{3}{2}\right) = \tfrac{3}{4}\sqrt{\pi}\,.$$

With the above definition, start with an integral value of n, say $n = 1$.

$$\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}} \frac{x}{\overline{|1}} = \frac{x^{\frac{1}{2}}}{\overline{|\frac{1}{2}}} = \frac{2x^{\frac{1}{2}}}{\sqrt{\pi}}$$

$$\frac{d^{\frac{3}{2}}}{dx^{\frac{3}{2}}} \frac{x}{\overline{|1}} = \frac{d}{dx} \frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}} \frac{x}{\overline{|1}} = \frac{d}{dx} \frac{2x^{\frac{1}{2}}}{\sqrt{\pi}} = \frac{1}{\sqrt{\pi x}}$$

$$\frac{d^{\frac{5}{2}}}{dx^{\frac{5}{2}}} \frac{x}{\overline{|1}} = \frac{d}{dx} \frac{d^{\frac{3}{2}}}{dx^{\frac{3}{2}}} \frac{x}{\overline{|1}} = \frac{x^{-\frac{1}{2}}}{-2\sqrt{\pi}}$$

Note that we started by differentiating fractionally. If in the last case we had started with two whole differentiations $\frac{d^{\frac{5}{2}}}{dx^{\frac{5}{2}}} \frac{x}{\overline{|1}}$ would have been zero.

Since $$\frac{d^2}{dx^2} \frac{x}{\overline{|1}} = \frac{d}{dx} \frac{x^o}{\overline{|o}} = o$$

Repeated fractional differentiations never give zero.
Find

$$\frac{d^{-\frac{1}{2}}}{dx^{-\frac{1}{2}}} \frac{x^n}{\overline{|n}} = \frac{x^{n+\frac{1}{2}}}{\overline{|n+\frac{1}{2}}}$$

for $n = 1$

$$\frac{d^{-\frac{1}{2}}}{dx^{-\frac{1}{2}}} \frac{x}{\overline{|1}} = \frac{x^{\frac{3}{2}}}{\overline{|\frac{3}{2}}} = \frac{3}{4}\sqrt{\pi}x^{\frac{3}{2}}$$

$$\frac{d^{-\frac{1}{2}}}{dx^{-\frac{1}{2}}} \frac{x^o}{\overline{|o}} = \frac{x^{\frac{1}{2}}}{\overline{|\frac{1}{2}}} = 2\sqrt{\frac{x}{\pi}}.$$

Similarly

$$\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}} \frac{x^o}{\overline{|o}} = \frac{x^{-\frac{1}{2}}}{\overline{|-\frac{1}{2}}} = \frac{1}{\sqrt{\pi x}}.$$

* See Janhke and Emde, pp. 26 and 31.

This last expression is the same as Heaviside obtained in comparing the heat problem with the electric problem.

This emphasizes the necessity of being careful about the zeros. There are zeros and zeros.

Take, for instance, successive derivatives of $\frac{t}{\lfloor 1}$.

$$\frac{d}{dt}\frac{t}{\lfloor 1} = \frac{t^0}{\lfloor 0}$$

$$\frac{d^2}{dt^2}\frac{t}{\lfloor 1} = \frac{t^{-1}}{\lfloor -1}$$

$$\frac{d^3}{dt^3}\frac{t}{\lfloor 1} = \frac{t^{-2}}{\lfloor -2}$$

$$\frac{d^n}{dt^n}\frac{t}{\lfloor 1} = \frac{t^{-n}}{\lfloor -n}$$

The last three expressions are usually written as zero since factorial of a negative integer is ∞. However, when operated upon with $\frac{p^{\frac{1}{2}}}{dt^{\frac{1}{2}}}$ they give $\frac{t^{-1\frac{1}{2}}}{\lfloor -1\frac{1}{2}}$, $\frac{t^{-2\frac{1}{2}}}{\lfloor -2\frac{1}{2}}$ and $\frac{t^{-n-\frac{1}{2}}}{\lfloor -n-\frac{1}{2}}$, respectively. All three are definite and different.

The application of fractional differentiation and integration will probably be extensive when mathematicians become interested in them. There is probably nothing mysterious about them, though at present it is difficult to find some physical significance similar to that of ordinary derivatives.

In conclusion it is well to again call the reader's attention to the fact that the solutions of the various problems have always been found by the operational method. Differentiations and integrations in the ordinary sense have not been resorted to.

The operators $p\,1$ and $\frac{1}{p}\,1$ have very definite properties.

One is the inverse operation of the other. We have not been handicapped by the fact that the unit function has a sharp corner, which if p were $\frac{d}{dt}$ would have caused endless discussion.

We have been able to get results with "discontinuous" functions, whatever these are. It has not been necessary to use Fourier's series and integrals with complications of Gibbs' phenomenon, etc.

Yet it must be admitted that, if with the present accepted definitions of functions, derivatives, etc., it had been possible to reconcile Heaviside's operators with conventional mathematics, a great contribution would have been made. But it can never be done.

In a paper to be presented soon before The Franklin Institute, Dr. J. J. Smith will suggest some modifications in the present mathematical definitions which enable Heaviside's operations to be made analogous to conventional mathematics.

The Fine Structure of the Nitrogen, Oxygen and Fluorine Lines in the Extreme Ultra-violet. I. S. BOWEN and R. A. MILLIKAN. (*Phil. Mag.*, Aug., 1924.)—With "a new 4-inch grating of very unusual quality, ruled for us at the Mt. Wilson Observatory," and having 1 m. focus, photographs have been obtained on Schumann plates up to and including the tenth order. This is truly an achievement.

In the previous work of these investigators they explored by vacuum grating spectroscopy ultra-violet wave-lengths down to 136 Å. "Our primary aim was to find what lines existed in this hitherto unexplored region, and it is altogether obvious, therefore, that we were purposely sacrificing resolution to intensity." They measured, at least approximately, the wave-lengths in the spectrum of the first order and with this to go on they now are in a position to recognize with certainty the same lines in the spectra of higher orders where their fine structure will manifest itself. From the fortunate proximity of certain lines in higher spectra to a well-known barium line, it has become possible to announce "that a set of wave-length standards correct to a few hundredths ångströms has been established in the region below 1000 Å., where no such standards have hitherto existed."

Not alone from the experimental side is this paper fruitful. Its contribution to theory is no less important. "We have found by a study with high resolution like that shown in the plate, of the fine structure of the doublets and triplets in our extreme ultra-violet spectra, that the X-ray doublet laws are applicable to the whole field of optics as well as to that of X-rays. We have also obtained unimpeachable proof that our 'hot spark' produces a whole series of stripped atoms, L_I, Be_{II}, B_{III}, C_{IV}, N_V; also Na_I, Mg_{II}, Al_{III}, Si_{IV}, P_V, Si_{VI}, as well as other series of similar atomic structure like B_I, C_{II}, N_{III}, O_{IV}. Indeed we have definitely identified lines due to all of the foregoing atoms." G. F. S.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

FORMULAS, TABLES, AND CURVES FOR COMPUTING THE MUTUAL INDUCTANCE OF TWO COAXIAL CIRCLES.[1]

By Harvey L. Curtis and C. Matilda Sparks.

[ABSTRACT.]

In this paper formulas for determining the mutual inductance of coaxial circles are derived and stated in equations which are convenient for computation. To accompany these formulas, tables and charts have been prepared to aid the computer. Values of the mutual inductance obtained from the charts will seldom be in error by more than 1 per cent. Values accurate to one-tenth of 1 per cent. can, in most cases, be obtained by the use of the tables. For determinations of the highest precision the formulas must be employed.

The charts and tables are also applicable to the inverse problem, namely, the determination of the dimensions of two coaxial circles which will give a desired mutual inductance. Approximate values of the dimensions are readily determined from the charts. By using two dimensions as given by the charts, a more accurate value of the third dimension can be obtained by using the tables.

The mutual inductance between two coaxial circles is a function of the measured dimensions; that is, the radii of the two circles and the distance between their centres. Two formulas are derived which give the mutual inductance in terms of the measured dimensions. These are

(1) $M = \frac{a^2}{A} P$ (circles close together)

(2) $M = \frac{a^2 A^2}{D^3} S$ (circles far apart)

Where

A is the radius of the larger circle,
a is the radius of the smaller circle,
D is the distance between the centres of the circles,

and

P and S are functions of a/A and A/D.

* Communicated by the Director.

[1] Scientific Papers, No. 492. Price, fifteen cents.

Double-entry tables are given by which the value of P (or S) can be determined for any given values of a/A and A/D. This is usually a very simple method, but in cases where the differences in the tables are large, interpolation may become inaccurate. In such cases, some other method should be used.

The number of parameters required to express the mutual inductance of two coaxial circles can be reduced to two by a proper choice of variables. The two parameters used in this paper are r_1 and k^2, where

$$r_1 = \sqrt{(A+a)^2 + D^2} \quad \text{and} \quad k^2 = \frac{4aA}{(A+a)^2 + D^2}$$

Then $M = r_1 H$ where H is a function of k^2. A single-entry table is sufficient to give the values of H in terms of k^2. However, interpolation in this table is not satisfactory when the coils are very close together or when they are far apart. When the coils are close together H is tabulated as a function of colog $(1 - k^2)$. When the coils are far apart, H is tabulated as a function of k^4. The interpolation in both of these tables is approximately linear.

A complete graphical representation of the mutual inductance of two coaxial circles in terms of their dimensions would require a chart in four-dimensional space. However, by using for variables the ratio of the dimensions, it is possible to make a satisfactory representation by means of a surface in three-dimensional space. The charts of this paper give the contour lines of this surface as projected on the coördinate planes.

INFLUENCE OF SULPHUR, OXYGEN, COPPER, AND MANGANESE ON THE RED-SHORTNESS OF IRON.[3]

By J. R. Cain.

[ABSTRACT.]

THIS investigation was undertaken in order to throw light on some disputed matters as to the elements in iron which cause it to be brittle when worked on a forge or otherwise above a red heat. Sulphur is generally admitted to be a cause of this defect, but data as to the minimum percentage of this element necessary before red-shortness disappears are rather meagre. Oxygen has been considered by many metallurgists to be as important in this regard as sulphur. The opinions on effect of copper

[3] Technologic Papers, No. 261. Price, ten cents.

are quite variable. Manganese is conceded generally to be a corrective for the red-shortness caused by sulphur and is thought by many to prevent or help prevent the red-shortness supposed to be caused by oxygen. Data as to the amounts of manganese to be used are given in the literature, but are in some cases contradictory.

Much of the discrepancy in views regarding effects of some of these elements is due to studies having been made on commercial steels in which it is difficult to insure that the element or elements studied are the only important ones present. This investigation has, therefore, been made with electrolytic iron or commercially pure iron as raw material. The melts were small (900 g. approximately) and made under fairly good control in carbon helix-vacuum furnaces or in a high-frequency induction furnace under air. The copper added was over 99.9 per cent. pure and the manganese over 98 per cent. The carbon content of the samples in most cases did not exceed 0.06 per cent. The ingots were forged to one-half inch bars and then tested for red-shortness by bending back and forth over a blacksmith's anvil in a temperature range of 1100 to 500° C. Samples classed as free from red-shortness were those which stood such a test without breaking.

The conclusions of this study are:

(1) Sulphur is the principal element responsible for red-shortness. In order to prevent red-shortness in iron not more than 0.01 per cent. sulphur should be present.

(2) Oxygen in amounts up to 0.20 per cent. does not cause red-shortness in pure iron if the sulphur is below 0.01 per cent.

(3) Manganese may prevent red-shortness in iron when present to the extent of three times the sulphur percentage if the oxygen percentage is not above 0.04 per cent.

(4) The presence of considerable amounts of oxygen in irons (0.10 per cent. and above) tends to reduce the efficiency of manganese in preventing red-shortness. The hypothesis is advanced that this is because some of the manganese reported in such irons is present as oxide.

(5) Copper (0.05 to 0.5 per cent.) is of minor importance in its effect on red-shortness of pure iron, but in some of the specimens described in this paper it tended to decrease the red-shortness.

COMPARISON OF AMERICAN AND FOREIGN CLAYS AS PAPER FILLERS.[1]

By Merle B. Shaw and George W. Bicking.

[ABSTRACT.]

THERE are approximately from 200,000 to 300,000 tons of clay used annually in the paper industry of the United States. Considering that the larger portion of this amount is of foreign source and that the clay resources of the United States are to a great extent undeveloped, the investigation herein reported was made to ascertain the relative merits of American and foreign clays for use as paper fillers.

The term "clay" is applied to a variety of earthy substances, differing widely in their origin and composition and in many of their physical properties. The essential requirements of a paper-making clay are: Good color; low content of grit, mica, and other impurities; and uniformity. The first requirement must be possessed by the clay in its original state, but the percentage of impurities, such as grit and mica, can be lowered by washing and separation.

The paper section of the Bureau of Standards is equipped for making paper in a semi-commercial way and under practical mill conditions. The experimental paper-making equipment available for the work on clays consisted of a fifty-pound wood tub beater with manganese-bronze bars and plate, a small Jordan with iron bars, a four-plate screen, and a twenty-nine-inch Fourdrinier machine with wire thirty-three feet long and having two presses, nine fifteen-inch dryers, a small machine stack of seven rolls, and a reel.

Commercial soda and sulphite pulps, and eight representative clays, five American and three foreign, were used in this investigation. Preliminary runs established best method of handling clay and determined constant factors.

The comparative study included tests for the amount of clay retained in the paper, the quality of the paper produced, and those physical properties of the clay (grit, etc.) that might affect the paper-manufacturing processes. Measurements for clay retention included analyses of samples taken at thirteen different positions

[1] Technologic Papers, No. 262. Price, fifteen cents.

on the paper machine. The retention values obtained for eight clays (Nos. 1 to 5, domestic; Nos. 6 to 8, foreign) were: Using 20 per cent. clay, 0.620, 0.611, 0.636, 0.630, 0.633, 0.655, 0.642, and 0.655, respectively; using 30 per cent. (runs varying per cent. being made for only two clays), clay No. 3, 0.600, clay No. 6, 0.632; using 10 per cent. clay No. 3, 0.720, clay No. 6, 0.748; and, using 5 per cent., clay No. 3, 0.744, clay No. 6, 0.740.

Similar tests were also made on samples taken by the authors from the paper machine of a commercial mill in operation. The retention data agree with the results obtained in duplicate work in the experimental mill of the bureau. The commercial-mill test also included a study of white-water losses. Determinations made during the seven and one-quarter-hour test period show that, on a 24-hour basis, without a save-all there would have been a loss of 3310 pounds of clay and 2369 pounds of pulp. By using the save-all there would be a loss of only 168 pounds of clay and 31 pounds of pulp during that interval. The save-all not only increased stock recovery, by permitting the reuse of water with high concentration of stock for dilution purposes, but also effected a saving of thousands of gallons of water during the twenty-four hours.

Measurements made on physical properties of the clays failed to show any correlation between such properties and the amount of retention in paper, but the color and grit tests slightly favored the foreign clays, although not sufficiently to justify the consideration of only these properties in the selection of clays. Physical tests made on the finished paper (bursting strength, finish, etc.) showed the results to be independent of the kind of clay added.

UNITED STATES GOVERNMENT MASTER SPECIFICATION FOR LIGHT WEIGHT DUCK (ARMY DUCK), GRAY.[1]

[ABSTRACT.]

This specification was prepared by the Cotton Duck Association and the technical committee on textiles of the Federal Specifications Board, under the supervision of the textile section

[1] Circular No. 166. Price, five cents.

of the Bureau of Standards. The specification includes weave, thread count and ply, width, length of bolt or roll, weight, and breaking strength. The standard grab method, 1 by 1 by 3 inches, is specified.

The ducks specified include 7, 8, 9, 10, 12, and 15 ounce per linear yard ducks of 28½-inch width, ranging in breaking strength (grab method) from 150 to 315 pounds in the warp and 85 to 200 pounds in the filling.

Absorption of Vapors of Various Liquids by Cotton. R. C. BRIMLEY. (*Nature*, Sept. 20, 1924.)—"Samples of bleached and unbleached two-fold Egyptian cotton hosiery yarn were suspended, at room temperatures, in a bottle, in the bottom of which was a layer of the liquid under consideration. The cotton (which before enclosure had been dried for four hours in a water oven and then weighed) was withdrawn, with suitable precautions, at intervals of several hours and weighed." The results are given in percentages, presumably of the initial weight of the cotton used. Water and glacial acetic acid were absorbed to a much greater extent than were the other liquids used. The unbleached cotton, saturated with their vapors, in both cases weighed from 18 to 20 per cent. more than at first. The bleached material absorbed about 1 per cent. more water and 1 per cent. less acid. Absolute alcohol, carbon disulphide, benzene, ether, nitrobenzene and acetone all are absorbed less than half as much as the first two named liquids. The amounts of CS_2, benzene, ether and nitrobenzene, taken up by the bleached and the unbleached fibres, are practically the same, while bleached cotton absorbs much more alcohol and acetone than does the unbleached yarn. G. F. S.

A Criticism of the Relativity Theory. G. VON GLEICH. (*Zeit. f. Physik*, Vol. 25, No. 3.)—Einstein's Theory of Relativity is so generally assumed to be accepted that it is well to call to mind now and then that perhaps the last word has not yet been said. In the United States Charles Lane Poor does not accept it and abroad there are those who are far from being convinced of its validity. This writer maintains that, while the theory came from the experiments of Michelson and Morley, its real connection with this physical result depends on a purely mathematical interpretation in which an assumption was made that is responsible for the relativistic conception of time. He further believes that all the other physical phenomena whose explanation is made by the theory—motion of the planet, Mercury, deviation of light that has passed near the sun and shifting toward the red of light from the sun—can be accounted for quite as well by other means. G. F. S.

NOTES FROM THE U. S. BUREAU OF CHEMISTRY.*

RELATION BETWEEN TOXICITY OF COTTON-SEED AND ITS GOSSYPOL CONTENT.[1]

By Erich W. Schwartze and Carl L. Alsberg.

[ABSTRACT.]

A COMPARISON of the toxicity of pure gossypol with that of four varieties of cotton-seed kernels was made in the Bureau of Chemistry. The results obtained indicate that the toxicity of cotton-seed kernels is due to their gossypol content.

Inasmuch as the gossypol content of cotton-seed kernels varies in the different producing regions, the regional variation in the toxicity of raw cotton-seed kernels is explained. This tendency to geographic variation in the toxicity of the raw material would be expected to affect the toxicity of cotton-seed press cake and meal made in different regions by the same manufacturing process. This conclusion is confirmed by the fact that in the Atlantic Coast states, where the gossypol content of cotton-seed averages slightly less than 1 per cent., cotton-seed poisoning is more feared than in the Southwest, where gossypol occurs in cotton-seed in smaller quantities.

PHARMACOLOGY OF GOSSYPOL.[2]

By Erich W. Schwartze and Carl L. Alsberg.

[ABSTRACT.]

PURE gossypol is capable of producing nearly all the manifestations reported as characteristic of the cotton-seed intoxication of farm animals. Therefore the conclusion seems warranted that gossypol is the principal causative agent of cotton-seed poisoning. This conclusion is supported by the fact that the toxicity of raw cotton-seed kernels varies almost directly with their gossypol content, or is only slightly less. Thus there can not be much of any

* Communicated by the Chief of the Bureau.

[1] Published in *J. Agr. Res.*, 28 (April 12, 1924) : 173.

[2] Published in *J. Agr. Res.*, 28 (April 12, 1924) : 191.

other toxic substance in cotton-seed kernels unless it be some decomposition product of gossypol. The pharmacology of such products has not been investigated.

STUDIES IN THE ANALYTICAL CHEMISTRY OF DRUGS. THE DETERMINATION OF TOTAL ALKALOIDS IN NUX VOMICA PREPARATIONS.[3]

By Samuel Palkin and H. R. Watkins.

[ABSTRACT.]

THE steps involved in the extraction, purification, and determination of the total alkaloids in nux vomica preparations have been studied experimentally with the view to a simplification of the method of assay.

The following conclusions have been reached: (*a*) The extreme slowness with which the last portions of alkaloids are extracted by the solvent is probably due to the presence of some alkaloid other than strychnine or brucine; (*b*) the extraction can be facilitated by careful control of the ammonia used; and (*c*) the essential purification can be effected by a simple preliminary precipitation. The latter step incidentally reduces the tendency for emulsion.

A shorter method is described and tables showing comparative results so obtained are given.

THE IDENTIFICATION OF PHENOLS BY MEANS OF THE SPECTROSCOPE.[4]

By Samuel Palkin and H. Wales.

[ABSTRACT.]

ABSORPTION spectrum examination, as a supplement to color tests, has a diagnostic value.

Absorption curves and tables of absorption-spectrum maxima for azo dyes prepared from certain phenols, by coupling with azo-*p*-nitrobenzene, have been worked out for the detection and identification of small quantities of phenols. A method adapted particularly to the examination of medicinal preparation has been developed in the Bureau of Chemistry.

[3] Published in *J. Am. Pharm. Assoc.*, 13 (August, 1924): 691.

[4] Published in *J. Am. Chem. Soc.*, 46 (June, 1924): 1488.

NOTES FROM THE U. S. BUREAU OF MINES.*

TENTH SEMI-ANNUAL MOTOR GASOLINE SURVEY.

By N. F. LeJeune and B. A. Landry.

For a number of years the Bureau of Mines has conducted semi-annual surveys to determine the changes in the characteristics of the motor gasoline being sold in the United States.

On the whole, it seems that a better grade of gasoline is now being marketed than was marketed during the same season a year ago. In comparison with the July, 1923, survey, the averages for the nine districts show that there was an appreciable decrease in the initial boiling point, a decrease in the end point and 20 per cent. point, and an increase in the 90 per cent. point.

As compared with the survey of January, 1924, the seasonal variation from winter to summer grades is not so marked, although there is a slight increase in the averages for initial boiling point. The average for "first drop" for the New York samples has increased, but the rest of the distillation range has remained practically unchanged. A slight increase is indicated in the Washington averages, with the exception of the 90 per cent. point, which decreased somewhat. The averages for Bartlesville, Pittsburgh, and San Francisco registered an increase throughout the entire distillation range except at the 90 per cent. point for San Francisco. The averages for Pittsburgh, Chicago, New Orleans, Bartlesville, and Denver show a noticeable increase in the 90 per cent. point and this increase in some cases is high enough to fail Federal Specifications for the 90 per cent. point. Tabulated results are presented in Serial 2636, recently issued by the bureau.

IMPROVEMENT OF THE GEOPHONE BY THE USE OF ELECTRICAL SOUND AMPLIFIERS.

By Walter T. Ackley, Jr., and Clifton M. Ralph.

The geophone has occupied a prominent position as a mine signalling device since its first introduction. The bureau has carried out tests at different times with a view to improving the early

* Communicated by the Director.

geophones. More recently, experiments were conducted in coöperation with the University of Pittsburgh, to adopt the vacuum-tube audio-frequency amplifier to a new form of geophone. While the results are not sufficient to warrant the drawing of definite conclusions as to the practicability of the electrical geophone, they indicate the possibility of such use. Because of the limited time available to the authors, only one geophone with its auxiliary amplifier was constructed. As is well known, two geophones are necessary in order to determine the direction of a sound. Although the principles upon which the electrical geophone operates appear to be fundamentally correct, it is not possible to state whether two such devices will function as efficiently (as regards determining direction of sound), as the present geophone. In order to operate efficiently as a direction finder, the amplifiers of the electrical geophone must each amplify with exactly the same degree of magnitude. By proper design and adjustment this can probably be accomplished without greatly complicating the design. Inasmuch as audio-frequency amplification is used, no tuning is necessary as is the case with tuned radio-frequency circuits. The same filament and plate batteries can be used to operate the amplifiers; and by the use of dry-cell tubes, requiring small filament and plate currents, it is believed that a compact, portable and practical instrument can be evolved. Further details are given in Serial 2639 of the bureau.

Salts in Oil Smoke Injure Slate Roof.—Salts contained in the soot from an oil burner were found by the Bureau of Standards to be the cause of extensive decay in the slate roof of the building in which the burner was housed. These salts, being soluble in water, were leached into the slate by the rains and were recrystallized in dry weather. The formation of the crystals tends to pry the particles of slate apart and produces an effect similar to frost action, only much more severe.

The disintegration of the slate was first noticed about six years ago and in some parts of the roof it has progressed to the danger point. It manifests itself only on the interior and is invisible from the outside. The process begins at the edge of the shingles and progresses along the direction of the cleavage, reducing the slate to powder.

THE FRANKLIN INSTITUTE.

(Proceedings of the Stated Meeting held Wednesday, October 15, 1924.)

HALL OF THE FRANKLIN INSTITUTE,
PHILADELPHIA, October 15, 1924.

President DR. WM. C. L. EGLIN *in the Chair.*

Additions to membership since last report, 26.

The Board of Managers submitted a revision of the By-Laws of the Institute with favorable recommendations.

Dr. James Barnes, on behalf of the Committee on Science and the Arts, introduced Dr. Harvey Fletcher, of New York City, to whom was awarded the Louis E. Levy Medal for his paper on "Physical Measurements of Audition and Their Bearing upon the Theory of Hearing," published in the JOURNAL of the Institute during 1923.

Mr. H. H. Quimby introduced Mr. Milton Roy Sheen, who was awarded the Edward Longstreth Medal for his tunnelling machine.

The Acting Secretary presented Mr. Thomas C. McBride, to whom was awarded the Edward Longstreth Medal for his invention of a locomotive feed water heater.

The medals were presented to the recipients by the President.

Prof. Archibald Vivian Hill, O.B.E., D.Sc., F.R.S., Jodrell Professor of Physiology, University College, London, presented the paper of the evening on "The Ultimate Nature of the Muscular Machine." The speaker described apparatus consisting of thermo-couples and delicate deflection apparatus capable of indicating differences of 0.000001° Centigrade, by means of which he has followed the energy changes during the phases of muscular contraction and relaxation lasting in all only a fraction of a second. From these experiments he has made important deductions concerning the thermodynamics of the process and the nature of the mechanism concerned. The subject was illustrated by lantern slides.

After brief discussion the meeting adjourned.

GEO. A. HOADLEY,
Acting Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, October 1, 1924.)

HALL OF THE INSTITUTE,
PHILADELPHIA, October 1, 1924.

DR. JAMES BARNES, *in the Chair.*

The following report was presented for final action:

No. 2822: The Scientific Work of Dr. John A. Anderson of the Mt. Wilson Solar Observatory, California, in Improvements in the Ruling of Gratings and in the Invention of a New Form of Seismograph.

The high degree of precision obtained by Professor Rowland in the ruling of gratings was, on his death, in danger of being lost since there was no

one who had the necessary experience in the manipulation of the Rowland ruling engine.

Anderson, who received the degree of Doctor of Philosophy from Johns Hopkins in 1907, attacked the problem of eliminating the uncertainties due to end-thrust and by the application of interference methods produced screws of a high degree of perfection.

By applying a similar test he succeeded in making the end bearings coaxial and determined the warp of the screw.

He also made a study of the ruling point and by means of improvements succeeded in producing gratings heretofore unattainable with respect to regularity of spacing and uniformity of groove.

The Anderson seismograph is of a much simpler and more portable form than those previously used. It consists essentially of a fine tungsten wire, kept vertical under suitable tension, and having a copper cylinder 20 mm. long and 2 mm. in diameter soldered against the side of the wire, which is so suspended as to be in the field of a controlling magnet. A small mirror is mounted on the copper wire and a beam of light being thrown onto this mirror is reflected onto a revolving drum upon which the vibrations are photographically recorded.

The portability and low cost of this seismograph are such as to make it possible to use it in locations from which the stationary and more expensive instruments are excluded. The Howard N. Potts medal was awarded to Doctor Anderson for his inventions.

The following report was presented for first reading:

No. 2811: Hob and Method of Making it.

GEO. A. HOADLEY,
Acting Secretary.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, October 8, 1924.)

RESIDENT MEMBERS.

MR. W. A. DOBSON, Naval Architect, 5918 Wayne Avenue, Philadelphia, Pennsylvania.

MR. WILLIAM G. ELLIS, Engineer, 4549 Morris Street, Germantown, Philadelphia, Pennsylvania.

MR. CHARLES E. INGERSOLL, Lawyer, 717 Lafayette Building, Fifth and Chestnut Streets, Philadelphia, Pennsylvania.

MR. H. H. MURRAY, Mechanical Engineer, 204 Lippincott Avenue, Riverton, New Jersey.

MR. J. F. OAKLEAF, Chief of Research Laboratories, Vacuum Oil Company, Paulsboro, New Jersey.

DR. RALPH PEMBERTON, Physician, 2120 Sansom Street, Philadelphia, Pennsylvania.

MR. A. RAYMOND RAFF, Builder, 1300 Oak Lane Avenue, Philadelphia, Pennsylvania.

MR. HOWARD E. REEDER, Civil Engineer, 542 East Mayland Street, Philadelphia, Pennsylvania.

MR. NICHOLAS G. ROOSEVELT, Vice-president, Day and Zimmermann, Incorporated, Engineers, 1600 Walnut Street, Philadelphia, Pennsylvania.

NON-RESIDENT MEMBERS.

DR. KARL K. DARROW, Physicist, 610 West One Hundred and Eleventh Street, New York City, New York.

NON-RESIDENT LIFE MEMBERS.

MR. FREDERICK SNARE, Contracting Engineer, 114 Liberty Street, New York City, New York.

CHANGES OF ADDRESS.

MR. MERRITT H. ADAMS, 373 Madison Avenue, Flushing, New York.

MR. ROBERT ATKINSON, Newtown, Pennsylvania.

MR. CARL G. BARTH, Boyne City, R. D. 2, Michigan.

MR. C. E. DAVIS, care of Indianapolis Water Company, P. O. Box 855, Indianapolis, Indiana.

PROFESSOR MATTHEW L. FORTIER, College of the Holy Cross, Worcester, Massachusetts.

MR. ANTHONY DE H. HOADLEY, 48 Shepard Street, Cambridge, Massachusetts.

MR. J. C. JOHNSON, General Superintendent of Telegraph, Pennsylvania Railroad Company, 812 Fox Building, Philadelphia, Pennsylvania.

MR. ALLEN I. MYERS, 314 North Chester Road, Swarthmore, Pennsylvania.

DR. A. D. POWER, Department of Physics, Lawrence College, Appleton, Wisconsin.

MR. DWIGHT P. ROBINSON, Stuyvesant Avenue, Milton Point, Rye, New York.

MR. FRANK W. SUTTON, Box 26, San Diego, California.

MR. J. E. SYMONS, 77 Davis Street, Westville, New Haven, Connecticut.

MR. ALFRED J. VOLLRATH, 158 Carpenter Lane, Germantown, Philadelphia, Pennsylvania.

MR. WM. R. WARNER, JR., Union League, Philadelphia, Pennsylvania.

MR. EARL H. WELTZ, 1311 South Fifty-second Street, Philadelphia, Pennsylvania.

MR. ALAN WOOD, 3RD, 109 County Line, Bryn Mawr, Pennsylvania.

NECROLOGY.

Mr. Charles William Masland, one of the most active members of The Franklin Institute, died, after a short illness, on September 15, 1924. He was a member of the extensive manufacturing firm of C. H. Masland and Sons, and was also connected with several other business enterprises.

His public spirit led him to become active in various civic organizations, both local and municipal, in which his ability and influence were of great value.

He was one of the most prominent laymen of the Methodist Church, and was widely known among all denominations for his high Christian character and liberality.

He joined The Franklin Institute on April 12, 1902, and was made

a member of the Committee on Science and the Arts in 1913. He became its Chairman in 1921, and served actively on this Committee until his death. During his connection with the Committee on Science and the Arts, he rendered service as a member and as Chairman of numerous Sub-committees; his work always being distinguished by his clear thinking, fidelity and thoroughness.

Mr. William H. Harding, 1127 Land Title Building, Philadelphia, Pennsylvania.

Mr. Charles Wirt, 508 East Washington Lane, Germantown, Philadelphia, Pennsylvania.

Mr. Henry R. Towne, 420 Park Avenue, New York City, New York.

Mr. Patrick B. Delany, South Orange, New Jersey.

LIBRARY NOTES.

PURCHASES.

Allen's Commercial Organic Analysis. Ed. 5, Vol. 2. 1924.

American Concrete Institute.—Proceedings of the Twentieth Annual Convention. 1924.

BARTON, EDWIN H.—Analytical Mechanics. Ed. 2. 1924.

BERNOULLI, JOHANN.—Die Differentialrechnung. (Ostwald's Klassiker der exakten Wissenschaften Nr. 211.) 1924.

HARVEY, LEONARD C.—Pulverized Fuel, Colloidal Fuel, Fuel Economy, and Smokeless Combustion. 1924.

HENRY, THOMAS ANDERSON.—The Plant Alkaloids. Ed. 2. 1924.

Journal für Praktische Chemie.—Sach- und Autorenregister zur neuen Folge Bd. 51–100, 1895–1920. 1924.

PARTINGTON, J. R., and SHILLING, W. G.—Specific Heats of Gases. 1924.

PORTER, HORACE C.—Coal Carbonization. 1924.

WERNER, ALFRED.—Beitrag zur Konstitution anorganischer Verbindungen. (Ostwald's Klassiker der exakten Wissenschaften Nr. 112.) 1924.

GIFTS.

American Society for Testing Materials, Year Book, August, 1924. Philadelphia, Pennsylvania, 1924. (From the Society.)

American Society of Civil Engineers, Transactions, Vol. 87, 1924. New York City, New York, 1924. (From the Society.)

Amsler, Alfred J., and Company, Abridged Catalogue E 25. Schaffhouse, Switzerland, 1924. (From the Company.)

Bristol Company, Catalogue No. 1401, Pyrometers. Waterbury, Connecticut, 1922. (From the Company.)

Canada Geological Survey, Memoir 139; Summary Report, 1923, Part A; Report of the Canadian Arctic Expedition, 1913–1918, Vol. xi: Geology and Geography. Ottawa, Canada, 1924. (From the Survey.)

Carnegie Endowment for International Peace, Year Book, 1924, No. 13. Washington, District of Columbia, 1924. (From the Endowment.)

Columbia University, Bulletin of Information, Schools of Mines, Engineering and Chemistry, 1924–1925. New York City, New York, 1924. (From the University.)

Defender Automatic Regulator Company, Catalogue No. 19, Defender Automatic Combustion Control for Fuel Saving in the Boiler Room. St. Louis, Missouri, no date. (From the Company.)

Dixon, Joseph, Crucible Company, Graphite, 1923, Vol. 25. Jersey City, New Jersey, no date. (From the Company.)

Dodds, Alexander, Company, Catalogue, 1923, Special Wood Working Machinery. Grand Rapids, Michigan, no date. (From the Company.)

Eastman Kodak Company, Abridged Scientific Publications from the Research Laboratory, Vol. 7, 1923. Rochester, New York, 1924. (From the Company.)

Gratz College, Register, 1924–1925. Philadelphia, Pennsylvania, 1924. (From the College.)

Institution of Automobile Engineers, Report of the Empire Motor Fuels Committee, Vol. 18, Part I. London, England, no date. (From the Institution.)

Institution of Engineers, Australia, Transactions, Vols. I and II, 1920 and 1921; Quarterly Bulletin, Vol. I, Nos. 1, 2 and 3. Sydney, Australia, 1921, 1923 and 1924. (From the Institution.)

Iowa Geological Survey, Vol. 29, Annual Reports, 1919 and 1920. Des Moines, Iowa, no date. (From the Survey.)

Iron and Steel Institute, Journal, No. 1, 1924, Vol. 109; Carnegie Scholarship Memoirs, Vol. 13. London, England, 1924. (From the Institute.)

John Carroll University, Catalogue, 1923–1924. Cleveland, Ohio, 1924. (From the University.)

Liverpool Engineering Society, Transactions, Vol. 44. Liverpool, England, 1923. (From the Society.)

Manchester Steam Users' Association, Memorandum by Chief Engineer for the Year 1923. Manchester, England, 1924. (From the Association.)

Marconi, Guglielmo, Results Obtained over Very Long Distances by Short Wave Directional Wireless Telegraphy, More Generally Referred to as the Beam System, Read at a Meeting of the Royal Society of Arts, July 2, 1924. London, England, 1924. (From the Author.)

Michigan College of Mines, Year Book, 1923–1924. Houghton, Michigan, 1924. (From the College.)

Morrell, Charles, The Squaration of the Circle and the Equalization. Chicago, Illinois, 1924. (From the Author.)

National Physical Laboratory, Report for the Year 1923. London, England, 1924. (From the Laboratory.)

New York City Board of Water Supply, Eighteenth Annual Report, 1923. New York City, New York, 1924. (From the Board.)

New York University, Announcement, 1924–1925, University and Bellevue Hospital Medical College. New York City, New York, 1924. (From the University.)

Norske Ingeniørforening, Aarbok, 1923–1924. Kristiania, Norway, 1924. (From the Society.)

Northampton Polytechnic Institute, Announcements, 1924–1925. London, England, no date. (From the Institute.)

Oglethorpe University, Catalogue, June, 1924. Oglethorpe University, Georgia, 1924. (From the University.)

Oregon Agricultural College, General Catalogue, 1924–1925; Experiment Station, Bulletins 204–206. Corvallis, Oregon, 1924. (From the College.)

Pennsylvania Railroad System, Bulletin No. 32, Tests of a Class Ils Freight Locomotive. Altoona, Pennsylvania, 1924. (From the System.)

Pennsylvania State Auditor-General, Biennial Report for the Two Years Ending May 31, 1923. Harrisburg, Pennsylvania, 1923. (From the State Librarian.)

Pennsylvania Water Supply Commission, Report, 1919–1920. Harrisburg, Pennsylvania, no date. (From the State Librarian.)

St. Joseph's College, Catalogue, 1923–1924. Philadelphia, Pennsylvania, no date. (From the College.)

St. Louis Water Commissioner, Annual Report for the Year Ending April 7, 1924. St. Louis, Missouri, 1924. (From the Commissioner.)

Tasmania Department of Mines, Geological Survey Bulletin No. 33 with Accompanying Maps; Bulletin No. 34; Underground Water Supply Paper No. 3; Report of the Secretary for Mines, 1923. Hobart, Tasmania, 1923 and 1924. (From the Department.)

University of Cape Town, General Prospectus, 1924. Cape Town, South Africa, 1923. (From the University.)

University of Cincinnati, Annual Catalogue, 1923–1924. Cincinnati, Ohio, 1924. (From the University.)

University of the State of New York, New York State Library, One Hundred and Sixth Annual Report, 1923. Albany, New York, 1924. (From the University.)

University of Utah, Catalogue, 1924–1925. Salt Lake City, Utah, 1924. (From the University.)

Verein deutscher Eisenhüttenleute Düsseldorf, Mitgliederverzeichnis, 1924. Düsseldorf, Germany, 1924. (From the Society.)

BOOK REVIEWS.

EINLEITUNG IN DIE MENGENLEHRE. By Adolf Fraenkel, of the University of Marburg. Second enlarged edition, 251 pages, 13 illustrations in the text, 8vo, paper bound. Berlin, Julius Springer, 1923. Price, $2.60.

The first edition of this work appeared in January, 1919, and therefore during the war. It was, it appears, written largely in the field and sent from there to be printed. The author expresses thanks to certain comrades who were not mathematicians who occupied dreary hours in assisting in the preparation of the work. The present edition, four years later than the first, appears, of course, under the conditions of nominal peace which now prevail in Europe.

The kind reception which was accorded to the first edition, both by mathematicians in general and by the critics, has justified the preparation of a second edition without material change in form or substance. Nevertheless the revision has been carefully made so as to care for all points that have developed since the first issue, and the preface calls particular attention to the changes made in paragraphs one to eleven.

The examples and the historical notices and the references to the literature have been materially increased and the name of Cantor, so intimately connected with the history of mathematics, becomes very prominent in this connection. The book has assumed considerably of the character of a mathematical text-book, though its original object was somewhat different.

A notable development in the work as contrasted with the first edition is the dealing with certain leading questions connected with the foundations of the theory and also in particular with the borderland between mathematics and philosophy. This phase of the problem, of breadth when the first edition was published, has increased considerably both from the mathematical and philosophical side, especially from the issuing in the last decade of the works of Brouwer, Hilbert, J. König, Weyl and others.

The subject of the work as the title indicates is an advanced department of mathematics. It exhibits great care and thoroughness in preparation and will no doubt find acceptance among those who pursue mathematics in its more abstruse phases. The mechanical execution of the book is, as is usual with the publishing firm, very excellent.

LA LUMIERE MONOCHROMATIQUES: SA PRODUCTION ET SON EMPLOI EN OPTIQUE PRACTIQUE. By Ch. Fabry. 38 pages, 8vo.

LES APPLICATIONS DES INTERFERENCES LUMINEUSES. By the same. 160 pages, 8vo. Paris, Editions de la *Revue d'Optique*, 1923.

These pamphlets are issues of the *Revue d'Optique theoretique et instrumentale*, devoted to Professor Fabry's researches on the application of light interference in the testing of surfaces. The employment of monochromatic light has given excellent results in his hands, and he devotes considerable space in the first pamphlet to the methods of obtaining such light. He gives preference to the mercury-arc lamp, but describes other apparatus in considerable detail. Striking photographs are given showing the value of the method. The monochromatic light represents a smaller range of waves, of course, than ordinary light and gives finer results. Many applications of the method to the investigations of surfaces are set forth, even the study of the surface of liquids is possible. In a footnote, Fabry deprecates the use of the symbol $\mu\mu$ for micro-micron, asserting that the additional Greek letter would indicate the millionth of a micron. There seems to be no logic in this. If μ indicates the one-thousandth of a millimetre, $\mu\mu$ should indicate the one-thousandth of the one-thousandth, that is the millionth of a millimetre. The author takes the proper abbreviation of a thousandth to be "m," but this seems to be a rather narrow view. Of the three great European nations that have led in scientific research, France seems to be the least cosmopolitan. The persistence with which French chemists continue to write exponents in chemical formulas as superiors when all the rest of the world uses inferiors, is an example of the curious chauvinism. For a long while they persisted in using Az for nitrogen and speaking of azotic acid, but this has been fortunately given up.

The two pamphlets contain a large amount of information and are well worth the study of those working in this field. It would probably be of value

to investigate whether ultra-violet and infra-red rays could be applied to such investigations. In such cases, of course, photography would be needed absolutely, as the phenomena are invisible to human vision.

HENRY LEFFMANN.

THE CHEMICAL ENGINEERING CATALOG. Ninth annual edition. Published by the Chemical Catalog Company, New York. Price, $10.

This edition, uniform in price and form with the preceding one that was reviewed in the issue of this JOURNAL for October, 1923, includes collected, condensed and standardized catalog data of equipment, machinery, laboratory supplies, heavy and fine chemicals and raw materials used in the industries employing chemical processes. The method of leasing the volume is still followed, but it may be purchased outright for the price noted above. The present issue, as might be expected, is somewhat larger than its immediate predecessor, including 1082 pages. The same editorial control has been in force as in the former issues, and the same care has been taken in securing correct and trustworthy information. A new departure is the introduction of a section on Directory data, giving the sources of building materials, and it is hoped that manufacturers of this class of equipment will be induced to secure space in the succeeding issues. The general character and certain special features were set forth in the review of the eighth edition, and it is merely necessary to say that the work affords excellent evidence of the high development to which American chemical industries have attained and is a comprehensive picture of the various phases and purposes of these industries.

HENRY LEFFMANN.

ABRIDGED SCIENTIFIC PUBLICATIONS OF THE RESEARCH LABORATORY OF THE EASTMAN KODAK COMPANY. Vol. vii, 1923. Pamphlet, 139 pages with indexes, illustrated, 8vo. Published by the Eastman Kodak Company.

This pamphlet, now established as an annual issue, contains more than a score of papers on special researches in photographic physics and chemistry, with some special descriptions of new forms of apparatus. It includes papers of recondite nature as well as purely practical ones available to the ordinary amateur. The Eastman Kodak Company have been doing a large amount of research work in many departments of photosensitiveness, and publication in this abridged form makes the data widely available. The work is largely devoted to the study of the problems and phenomena of the silver halide emulsion which is, of course, the one that dominates the photographic field, but is to be hoped that before long the laboratory will find time to study at some length the photosensitiveness of other metallic compounds. The sensitiveness of the mercurous salts might yield some interesting results. There is, indeed, a wide range of photosensitiveness, although few substances have as yet been found to have it in such degree as to be available for practical purposes.

HENRY LEFFMANN.

WIRELESS POSSIBILITIES. By A. M. Low. 77 pages, several illustrations, 18mo. New York, E. P. Dutton and Company, 1924. Price, $1.

This is one of a series entitled "To-day and To-morrow Series." It consists of a discussion of various lines in which the modern radio broadcasting may be developed. The author allows himself considerable freedom of imagi-

nation, but our experiences in the development of various modern discoveries lead us to believe that in the particular field to which this book is devoted very remarkable results, both interesting and valuable, may be obtained.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS. Report No. 190, Correcting Horsepower Measurements to a Standard Temperature, by Stanwood W. Sparrow. 14 pages, illustrations, quarto. Washington, Government Printing Office, 1924. Price, five cents.

The relation between the temperature of the air at the entrance to the carburetor and the power developed by the engine is discussed. Its scope is limited to a consideration of the range of temperatures likely to result from changes of season, locality, or altitude, since its primary aim is the finding of a satisfactory basis for correcting power measurements to a standard temperature.

The tests upon which this report is based were made upon aviation engines in the altitude laboratory of the Bureau of Standards. From the results of over 1600 tests it is concluded that if calculations be based on the assumption that the indicated horsepower of an engine varies inversely as the square root of the absolute temperature of the carburetor air the values obtained will check closely experimental measurements. The extent to which this relationship would be expected from theoretical considerations is discussed and some suggestions are given relative to the use of this relationship in correcting horsepower measurements.

Report No. 196, Comparison of Model Propeller Tests with Airfoil Theory, by William F. Durand and E. P. Lesley. 26 pages, illustrations, quarto. Washington, Government Printing Office, 1924. Price, ten cents.

The purpose of the investigation covered by the present report was the examination of the degree of approach which may be anticipated between laboratory tests on model airplane propellers and results computed by the airfoil theory, based on tests of airfoils representative of successive blade sections.

It is known that the corrections for angles of attack and for aspect ratio, speed and interference rest either on experimental data or on somewhat uncertain theoretical assumptions. The general situation as regards these four sets of corrections is far from satisfactory, and while it is recognized that occasion exists for the consideration of such corrections, their determination in any given case is a matter of considerable uncertainty. There exists at the present time no theory generally accepted and sufficiently comprehensive to indicate the amount of such corrections, and the application to individual cases of the experimental data available is, at best, uncertain.

While the results of this first phase of the investigation are less positive than had been hoped might be the case, nevertheless the establishment of the general degree of approach between the two sets of results which might be anticipated on the basis of this simpler mode of application, seems in any event to have been desirable and to have abundantly justified the time and effort required.

PUBLICATIONS RECEIVED.

The Chemistry of Enzyme Actions. Second and revised edition by K. George Falk. American Chemical Society Monograph Series. 249 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Incorporated, 1924.

Laboratory Experiments in Physics. By Elmer Reginald Drew, Ph.D., and Hermon W. Farwell, A.M. 143 pages, illustrations, 12mo. New York, The Century Company, 1924.

A Laboratory Manual of Machine Shop Practice. By Jerry H. Service, B.S., and George E. Frease. 106 pages, illustrations, 12mo. New York, D. Van Nostrand Company, 1924. Price, $1.25.

Colloid Chemistry. An introduction, with some practical applications by Jerome Alexander, M.Sc. Second edition, revised and enlarged. 208 pages, illustrations, 12mo. New York, D. Van Nostrand Company, 1924. Price, $2.

Freehand Drafting. By Anthony E. Zipprich, with an introduction by Carl L. Svensen, M.E. 131 pages, illustrations, 8vo. New York, D. Van Nostrand Company, 1924. Price, $1.60.

L'Origine Tourbillonnaire de l'atome et ses conséquences. Par Jean Varin d'Ainvelle. 215 pages, 8vo. Paris, Gauthier-Villars et Cie., 1924. Price, in paper, 20 Francs.

U. S. Coast and Geodetic Survey. Terrestrial Magnetism. Magnetic Declination in Arkansas in 1923, by W. N. McFarland, Mathematician, Division of Terrestrial Magnetism. 29 pages, plate, map, 8vo. Washington, Government Printing Office, 1923. Price, ten cents. Magnetic Declination in Florida in 1924, by W. N. McFarland, Mathematician, Division of Terrestrial Magnetism. 36 pages, plate, map, 8vo. Washington, Government Printing Office, 1924. Price, ten cents.

National Advisory Committee for Aeronautics. Technical Notes, No. 202, Experimental Investigation of the Effect of an Oscillating Airstream (Katzmayer effect) on the Characteristics of Airfoils, by Toussaint, Kerneis and Girault. 32 pages, illustrations, quarto. No. 203, A Short Method of Calculating Torsional Stresses in an Airplane Fuselage, by John E. Younger. 23 pages, photographs, illustrations, quarto. No. 204, A Study of Static Stability of Airships, by Frank Rizzo. 50 pages, illustrations, quarto. No. 205, The Logarithmic Polar Curve, Its Theory and Application to the Predetermination of Airplane Performance, by Val. Cronstedt. 20 pages, illustrations, quarto. Washington, Committee, 1924.

Western Electric Company, Engineering Department. Reprints: Photomicrography and Technical Microscopy, by Francis F. Lucas. 45 pages, illustrations, 8vo. Some Contemporary Advances in Physics—III, by Karl K. Darrow. 31 pages, illustrations, 8vo. The Relation between Thermal Emissive Power and Electrical Resistivity, by C. Davisson and J. R. Weeks, Jr. 25 pages, illustrations, 8vo. Audiometric Measurements and Their Uses, by Harvey Fletcher. 13 pages, illustrations, 8vo. A Note on the Thermodynamics of Thermionic Emission, by C. Davisson. 6 pages, 8vo. New York, Company, 1924.

The Organization and Functions of The Institute of Industrial Research of Lehigh University. Circular No. 1, Lehigh Institute of Research. 19 pages, 8vo. Bethlehem, Lehigh University, 1924.

National Electric Light Association. Serial Report of the Prime Movers Committee: Stokers and Furnaces. 45 pages, illustrations, quarto. Boilers, Superheaters and Economizers. 57 pages, illustrations, quarto. Design and Maintenance of a 7500 Kw. Central Station Plant. 13 pages, illustrations, quarto. Lubrication. 13 pages, illustrations, quarto. Condensing Equipment. 60 pages, illustrations, quarto. Pulverized Fuel. 32 pages, illustrations, quarto. New York, Association, 1924.

Heat from the Stars. (Explorations and Field-work of the Smithsonian Institution in 1923. *Smithsonian Mis. Coll.*, Vol. 76, No. 10.)—Doctor Abbot made an expedition to Mt. Wilson, if so formidable a name can be connected with so accessible a place. Three investigations were in view. That relating to the utilization of the sun's heat for cooking was halted by a fire due to leaking oil, but not before "oven temperatures up to 175° C. were reached." For the second investigation, which has to do with a possible connection between the quantity of ozone in the air and the amount of radiation arriving from the sun, the apparatus was set up, but the actual work was deferred till later. Ozone is formed by the action of ultra-violet light on oxygen, and occurs in appreciable quantities in the upper regions of the atmosphere. Fabry and Buisson, the former of whom was present at the recent Centenary Anniversary of The Franklin Institute, have devised photographic methods of determining the quantity of ozone present in the air, and have shown that there is enough of it in the upper air to cause practical extinction by absorption of the extreme ultra-violet rays in sunlight. This effect counts for little in meteorology but, in addition, ozone has a distinct absorption band in the infra-red. It thus prevents the escape into space of radiation emitted by the earth itself and in this way becomes a factor in the balance of power between the radiation received from the sun and that sent out by our planet.

Definite and valuable results were obtained from the third investigation, the quantity of heat received from the stars. A radiometer method of measurement was used, and the results were in accord with those previously obtained from the bolometer. "It is possible, in this way, by comparison with the sun, to determine the intensity of star heat nearly as accurately as we can determine the intensity of solar heat. From rough preliminary computations it appears, for instance, that the radiation sent by the bright star 'Aldebaran,' if collected over a square mile, would produce one calory of heat per minute, whereas the sun's radiation collected over a surface of one square centimetre, that is to say, about three-eighths of an inch on a side, amounts to 1.94 calories per square centimetre per minute. It is also possible, in this manner, to determine the diameters of some of the stars, providing their distance from the earth is known. In

the case of the star 'Aldebaran,' preliminary computations give the diameter as 58,000,000 miles." The radiation from this star fits almost precisely upon the curve of the radiation of the perfect radiator or "absolutely black body" of 3000° absolute centigrade. The fit is, indeed, startlingly close, so that one has no hesitation in assigning to the star "Aldebaran" the temperature 3000° absolute centigrade. G. F. S.

The Mass Spectra of Zirconium and Some Other Elements. F. W. ASTON. (*Nature*, Aug. 23, 1924.)—After many difficulties zirconium mass lines have been obtained at 90, 92, 94 and perhaps at 96. Cerium has been found to consist of a strong component 140 and a weak one 142. Neodymium has isotopes 142, 144, 146 and possibly 145.

How barium manages to have an atomic weight of 137.37 is still unknown; it shows a strong line at 138, but no component of mass number less than 136 can be detected. There is, in fact, a component at 136, but it is so faint that it cannot possibly by its admixture with the 138 component reduce the general atomic weight to 137.37.

The existence of an isotope of silicon having the weight 30 is confirmed. G. F. S.

THE FRANKLIN INSTITUTE

OFFICERS FOR 1924

President	W. C. L. EGLIN
Vice-presidents	HENRY HOWSON C. C. TUTWILER GELLERT ALLEMAN
Acting Secretary	GEORGE A. HOADLEY
Assistant Secretary and Librarian	ALFRED RIGLING
Treasurer	BENJAMIN FRANKLIN
Actuary	WILLIAM F. JACKSON, JR.

Board of Managers

GEORGE H. BENZON, JR.
CHARLES E. BONINE
FRANCIS T. CHAMBERS
G. H. CLAMER
THEOBALD F. CLARK
WALTON CLARK
CHARLES DAY
WALTON FORSTALL
CLARENCE A. HALL
ALFRED C. HARRISON
NATHAN HAYWARD
CHARLES A. HEXAMER
GEORGE A. HOADLEY
ROBERT W. LESLEY
RALPH MODJESKI
MARSHALL S. MORGAN
LAWRENCE T. PAUL
JAMES S. ROGERS
GEORGE D. ROSENGARTEN
E. H. SANBORN
HASELTINE SMITH
J. T. WALLIS
WILLIAM C. WETHERILL

Board of Trustees

JOHN HAMPTON BARNES, President
WILLIAM L. AUSTIN
CHARLES E. BRINLEY
WALTON CLARK
JOHN GRIBBEL
ALFRED C. HARRISON
FREDERIC ROSENGARTEN

PRESS OF
J. B. LIPPINCOTT COMPANY
PHILADELPHIA